Chapter (8)

# **Three Phase Induction Motors**

# Introduction

The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control. We usually prefer d.c. motors when large speed variations are required. Nevertheless, the 3-phase induction motors are simple, rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements. In this chapter, we shall focus our attention on the general principles of 3-phase induction motors.

# 8.1 Three-Phase Induction Motor

Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a "transformer-type" a.c. machine in which electrical energy is converted into mechanical energy.

#### Advantages

- (i) It has simple and rugged construction.
- (ii) It is relatively cheap.
- (iii) It requires little maintenance.
- (iv) It has high efficiency and reasonably good power factor.
- (v) It has self starting torque.

#### Disadvantages

- (i) It is essentially a constant speed motor and its speed cannot be changed easily.
- (ii) Its starting torque is inferior to d.c. shunt motor.

# 8.2 Construction

A 3-phase induction motor has two main parts (i) stator and (ii) rotor. The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.

#### 1. Stator

It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations [See Fig. (8.1)]. The insulated connected to form a balanced 3-phase star or delta connected



circuit. The 3-phase stator winding is wound for a definite number of poles as per requirement of speed. Greater the number of poles, lesser is the speed of the motor and vice-versa. When 3-phase supply is given to the stator winding, a rotating magnetic field (See Sec. 8.3) of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.

### 2. Rotor

The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types:

- (i) Squirrel cage type (ii) Wound type
- (i) **Squirrel cage rotor**. It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot. All these bars are joined at each end by metal rings called end rings [See Fig. (8.2)]. This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator.

Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of 3-phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque.



(ii) **Wound rotor**. It consists of a laminated cylindrical core and carries a 3phase winding, similar to the one on the stator [See Fig. (8.3)]. The rotor winding is uniformly distributed in the slots and is usually star-connected. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The three brushes are connected to a 3-phase star-connected rheostat as shown in Fig. (8.4). At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed.



The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.

# 8.3 Rotating Magnetic Field Due to 3-Phase Currents

When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do no remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating Held. It can be shown that magnitude of this rotating field is constant and is equal to 1.5  $\phi_m$  where  $\phi_m$  is the maximum flux due to any phase.

To see how rotating field is produced, consider a 2-pole, 3i-phase winding as shown in Fig. (8.6 (i)). The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as  $I_x$ ,  $I_y$  and  $I_z$  [See Fig. (8.6 (ii))]. Referring to Fig. (8.6 (ii)), the fluxes produced by these currents are given by:



$$\phi_x = \phi_m \sin \omega t$$
  
 $\phi_y = \phi_m \sin (\omega t - 120^\circ)$   
 $\phi_z = \phi_m \sin (\omega t - 240^\circ)$ 

r ig.(0.5)

Here  $\phi_m$  is the maximum flux due to any phase. Fig. (8.5) shows the phasor diagram of the three fluxes. We shall now prove that this 3-phase supply produces a rotating field of constant magnitude equal to 1.5  $\phi$ m.



(i) At instant 1 [See Fig. (8.6 (ii)) and Fig. (8.6 (iii))], the current in phase X is zero and currents in phases Y and Z are equal and opposite. The currents are flowing outward in the top conductors and inward in the bottom conductors. This establishes a resultant flux towards right. The magnitude of the resultant flux is constant and is equal to 1.5  $\phi_m$  as proved under:

**Fig.(8.7)** 

At instant 1,  $\omega t = 0^{\circ}$ . Therefore, the three fluxes are given by;

$$\phi_{x} = 0; \qquad \phi_{y} = \phi_{m} \sin(-120^{\circ}) = -\frac{\sqrt{3}}{2} \phi_{m};$$
  
$$\phi_{z} = \phi_{m} \sin(-240^{\circ}) = \frac{\sqrt{3}}{2} \phi_{m}$$

The phasor sum of  $-\phi_y$  and  $\phi_z$  is the resultant flux  $\phi_r$  [See Fig. (8.7)]. It is clear that:

Resultant flux, 
$$\phi_r = 2 \times \frac{\sqrt{3}}{2} \phi_m \cos \frac{60^\circ}{2} = 2 \times \frac{\sqrt{3}}{2} \phi_m \times \frac{\sqrt{3}}{2} = 1.5 \phi_m$$

(ii) At instant 2, the current is maximum (negative) in  $\phi_v$  phase Y and 0.5 maximum (positive) in phases X and Y. The magnitude of resultant flux is 1.5  $\phi_m$  as proved under:

120

**Fig.(8.8)** 

At instant 2,  $\omega t = 30^{\circ}$ . Therefore, the three fluxes are given by;

$$\phi_{x} = \phi_{m} \sin 30^{\circ} = \frac{\phi_{m}}{2}$$
$$\phi_{y} = \phi_{m} \sin (-90^{\circ}) = -\phi_{m}$$
$$\phi_{z} = \phi_{m} \sin (-210^{\circ}) = \frac{\phi_{m}}{2}$$

The phasor sum of  $\phi_x$ ,  $-\phi_y$  and  $\phi_z$  is the resultant flux  $\phi_r$ 

Phasor sum of  $\phi_x$  and  $\phi_z$ ,  $\phi'_r = 2 \times \frac{\phi_m}{2} \cos \frac{120^\circ}{2} = \frac{\phi_m}{2}$ Phasor sum of  $\phi'_r$  and  $-\phi_y$ ,  $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$ Note that resultant flux is displaced 30° clockwise from position 1. (iii) At instant 3, current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y arc  $0.866 \times \text{max}$ . value). The magnitude of resultant flux is 1.5  $\phi_m$  as proved under:



**Fig.(8.9**)

At instant 3,  $\omega t = 60^{\circ}$ . Therefore, the three fluxes are given by;

$$\phi_{x} = \phi_{m} \sin 60^{\circ} = \frac{\sqrt{3}}{2} \phi_{m};$$
  
$$\phi_{y} = \phi_{m} \sin(-60^{\circ}) = -\frac{\sqrt{3}}{2} \phi_{m};$$
  
$$\phi_{z} = \phi_{m} \sin(-180^{\circ}) = 0$$

The resultant flux  $\phi_r$  is the phasor sum of  $\phi_x$  and  $-\phi_y$  (b  $\phi_z = 0$ ).

$$\phi_{\rm r} = 2 \times \frac{\sqrt{3}}{2} \phi_{\rm m} \, \cos \frac{60^{\circ}}{2} = 1.5 \, \phi_{\rm m}$$

Note that resultant flux is displaced  $60^{\circ}$  clockwise from position 1.

(iv) At instant 4, the current in phase X is maximum (positive) and the currents in phases V and Z are equal and negative (currents in phases V and Z are  $0.5 \times \text{max}$ . value). This establishes a resultant flux downward as shown under:



**Fig.**(7.10)

At instant 4,  $\omega t = 90^{\circ}$ . Therefore, the three fluxes are given by;

$$\phi_{x} = \phi_{m} \sin 90^{\circ} = \phi_{m}$$
$$\phi_{y} = \phi_{m} \sin (-30^{\circ}) = -\frac{\phi_{m}}{2}$$
$$\phi_{z} = \phi_{m} \sin (-150^{\circ}) = -\frac{\phi_{m}}{2}$$

The phasor sum of  $\varphi_x, -\varphi_y$  and  $-\varphi_z$  is the resultant flux  $\varphi_r$ 

Phasor sum of  $-\phi_z$  and  $-\phi_y$ ,  $\phi'_r = 2 \times \frac{\phi_m}{2} \cos \frac{120^\circ}{2} = \frac{\phi_m}{2}$ 

Phasor sum of  $\phi'_r$  and  $\phi_x$ ,  $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$ 

Note that the resultant flux is downward i.e., it is displaced  $90^{\circ}$  clockwise from position 1.

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value (= 1.5  $\phi_m$ , where  $\phi_m$  is the maximum flux due to any phase).

#### Speed of rotating magnetic field

The speed at which the rotating magnetic field revolves is called the synchronous speed (N<sub>s</sub>). Referring to Fig. (8.6 (ii)), the time instant 4 represents the completion of one-quarter cycle of alternating current  $I_x$  from the time instant 1. During this one quarter cycle, the field has rotated through 90°. At a time instant represented by 13 or one complete cycle of current  $I_x$  from the origin, the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, fur P poles, the rotating field makes one revolution in P/2 cycles of current.

$$\therefore \qquad \text{Cycles of current} = \frac{P}{2} \times \text{revolutions of field}$$

or Cycles of current per second =  $\frac{P}{2}$  × revolutions of field per second

Since revolutions per second is equal to the revolutions per minute  $(N_s)$  divided by 60 and the number of cycles per second is the frequency f,

$$\therefore \quad f = \frac{P}{2} \times \frac{N_s}{60} = \frac{N_s P}{120}$$
$$N_s = \frac{120 f}{P}$$

or

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

#### **Direction of rotating magnetic field**

The phase sequence of the three-phase voltage applied to the stator winding in Fig. (8.6 (ii)) is X-Y-Z. If this sequence is changed to X-Z-Y, it is observed that direction of rotation of the field is reversed i.e., the field rotates counterclockwise rather than clockwise. However, the number of poles and the speed at which the magnetic field rotates remain unchanged. Thus it is necessary only to change the phase sequence in order to change the direction of rotation of the magnetic field. For a three-phase supply, this can be done by interchanging any two of the three lines. As we shall see, the rotor in a 3-phase induction motor runs in the same direction as the rotating magnetic field. Therefore, the

direction of rotation of a 3-phase induction motor can be reversed by interchanging any two of the three motor supply lines.

# 8.4 Alternate Mathematical Analysis for Rotating Magnetic Field

We shall now use another useful method to find the magnitude and speed of the resultant flux due to three-phase currents. The three-phase sinusoidal currents produce fluxes  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  which vary sinusoidally. The resultant flux at any instant will be the vector sum of all the three at that instant. The fluxes are represented by three variable magnitude vectors [See Fig. (8.11)]. In Fig. (8.11),



the individual flux directions arc fixed but their magnitudes vary sinusoidally as does the current that produces them. To find the magnitude of the resultant flux, resolve each flux into horizontal and vertical components and then find their vector sum.

$$\phi_{\rm h} = \phi_{\rm m} \cos t \, \omega t - \phi_{\rm m} \cos(\omega t - 120^{\circ}) \cos 60^{\circ} - \phi_{\rm m} \cos(\omega t - 240^{\circ}) \cos 60^{\circ}$$
$$= \frac{3}{2} \phi_{\rm m} \cos \omega t$$
$$\phi_{\rm v} = 0 - \phi_{\rm m} \cos(\omega t - 120^{\circ}) \cos 60^{\circ} + \phi_{\rm m} \cos(\omega t - 240^{\circ}) \cos 60^{\circ} = \frac{3}{2} \phi_{\rm m} \sin \omega t$$

The resultant flux is given by;

...

$$\phi_{\rm r} = \sqrt{\phi_{\rm h}^2 + \phi_{\rm v}^2} = \frac{3}{2}\phi_{\rm m} \left[\cos^2 \omega t + (-\sin \omega t)^2\right]^{1/2} = \frac{3}{2}\phi_{\rm m} = 1.5\phi_{\rm m} = \text{Constant}$$

Thus the resultant flux has constant magnitude (= 1.5  $\phi_m$ ) and does not change with time. The angular displacement of  $\phi_r$  relative to the OX axis is

$$\tan \theta = \frac{\phi_{v}}{\phi_{h}} = \frac{\frac{3}{2}\phi_{m} \sin \omega t}{\frac{3}{2}\phi_{m} \cos \omega t} = \tan \omega t$$
  
$$\Theta = \omega t$$
  
**Fig.(8.12)**

Thus the resultant magnetic field rotates at constant angular velocity  $\omega$ ( = 2  $\pi$ f) rad/sec. For a P-pole machine, the rotation speed ( $\omega_m$ ) is

$$\omega_{\rm m} = \frac{2}{\rm P}\omega \ \rm rad/sec$$

 $\frac{2\pi N_s}{60} = \frac{2}{P} \times 2\pi f \qquad \dots N_s \text{ is in r.p.m.}$  $\therefore \qquad N_s = \frac{120 \text{ f}}{P}$ 

Thus the resultant flux due to three-phase currents is of constant value (=  $1.5 \phi_m$  where  $\phi_m$  is the maximum flux in any phase) and this flux rotates around the stator winding at a synchronous speed of 120 f/P r.p.m.

For example, for a 6-pole, 50 Hz, 3-phase induction motor,  $N_{r} = 120 \times 50/6 = 1000 \text{ r.p.m.}$  It means that flux rotates around the stator at a speed of 1000 r.p.m.

# 8.5 Principle of Operation

Consider a portion of 3-phase induction motor as shown in Fig. (8.13). The operation of the motor can be explained as under:

- (i) When 3-phase stator winding is energized from a 3-phase supply, a rotating magnetic field is set up which rotates round the stator at synchronous speed  $N_s$  (= 120 f/P).
- (ii) The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are



**Fig.(1-)** 

stationary. Due to the relative speed between the rotating flux and the stationary rotor, e.m.f.s are induced in the rotor conductors. Since the rotor circuit is short-circuited, currents start flowing in the rotor conductors.

- (iii) The current-carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field.
- (iv) The fact that rotor is urged to follow the stator field (i.e., rotor moves in the direction of stator field) can be explained by Lenz's law. According to this law, the direction of rotor currents will be such that they tend to oppose the cause producing them. Now, the cause producing the rotor currents is the relative speed between the rotating field and the stationary rotor conductors. Hence to reduce this relative speed, the rotor starts running in the same direction as that of stator field and tries to catch it.

or

# 8.6 Slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the suitor field speed (N<sub>s</sub>). This difference in speed depends upon load on the motor.

The difference between the synchronous speed  $N_s$  of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.,

% age slip, 
$$s = \frac{N_s - N}{N_s} \times 100$$

- (i) The quantity  $N_s N$  is sometimes called slip speed.
- (ii) When the rotor is stationary (i.e., N = 0), slip, s = 1 or 100 %.
- (iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

### 8.7 Rotor Current Frequency

The frequency of a voltage or current induced due to the relative speed between a vending and a magnetic field is given by the general formula;

Frequency = 
$$\frac{NP}{120}$$

where N = Relative speed between magnetic field and the winding P = Number of poles

For a rotor speed N, the relative speed between the rotating flux and the rotor is  $N_s - N$ . Consequently, the rotor current frequency f is given by;

$$f' = \frac{(N_s - N)P}{120}$$

$$= \frac{s N_s P}{120} \qquad (b \ s = \frac{N_s - N}{N_s})$$

$$= sf \qquad (b \ f = \frac{N_s P}{120})$$

i.e., Rotor current frequency = Fractional slip x Supply frequency

(i) When the rotor is at standstill or stationary (i.e., s = 1), the frequency of rotor current is the same as that of supply frequency ( $f = sf = 1 \times f = f$ ).

(ii) As the rotor picks up speed, the relative speed between the rotating flux and the rotor decreases. Consequently, the slip s and hence rotor current frequency decreases.

*Note*. The relative speed between the rotating field and stator winding is  $N_s - 0 = N_s$ . Therefore, the frequency of induced current or voltage in the stator winding is  $f = N_s P/120$ —the supply frequency.

# 8.8 Effect of Slip on The Rotor Circuit

When the rotor is stationary, s = 1. Under these conditions, the per phase rotor e.m.f.  $E_2$  has a frequency equal to that of supply frequency f. At any slip s, the relative speed between stator field and the rotor is decreased. Consequently, the rotor e.m.f. and frequency are reduced proportionally to  $sE_s$  and sf respectively. At the same time, per phase rotor reactance  $X_2$ , being frequency dependent, is reduced to  $sX_2$ .

Consider a 6-pole, 3-phase, 50 Hz induction motor. It has synchronous speed  $N_s = 120 \text{ f/P} = 120 \times 50/6 = 1000 \text{ r.p.m.}$  At standsill, the relative speed between stator flux and rotor is 1000 r.p.m. and rotor e.m.f./phase =  $E_2(say)$ . If the full-load speed of the motor is 960 r.p.m., then,

$$s = \frac{1000 - 960}{1000} = 0.04$$

(i) The relative speed between stator flux and the rotor is now only 40 r.p.m. Consequently, rotor e.m.f./phase is reduced to:

$$E_2 \times \frac{40}{1000} = 0.04E_2$$
 or  $sE_2$ 

(ii) The frequency is also reduced in the same ratio to:

$$50 \times \frac{40}{1000} = 50 \times 0.04$$
 or sf

(iii) The per phase rotor reactance  $X_2$  is likewise reduced to:

$$X_2 \times \frac{40}{1000} = 0.04 X_2$$
 or  $sX_2$ 

Thus at any slip s,

Rotor e.m.f./phase =  $sE_2$ Rotor reactance/phase =  $sX_2$ Rotor frequency = sf

where  $E_2, X_2$  and f are the corresponding values at standstill.

#### 8.9 Rotor Current

Fig. (8.14) shows the circuit of a 3-phase induction motor at any slip s. The rotor is assumed to be of wound type and star connected. Note that rotor e.m.f./phase and rotor reactance/phase are s  $E_2$  and s $X_2$  respectively. The rotor resistance/phase is R2 and is independent of frequency and, therefore, does not depend upon slip. Likewise, stator winding values R<sub>1</sub> and X<sub>1</sub> do not depend upon slip.





At standstill. Fig. (8.15 (i)) shows one phase of the rotor circuit at standstill.



**Fig.**(8.15)

When running at slip s. Fig. (8.15 (ii)) shows one phase of the rotor circuit when the motor is running at slip s.

Rotor current, 
$$I'_2 = \frac{sE_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Rotor p.f., 
$$\cos \phi'_2 = \frac{R_2}{Z'_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

# 8.10 Rotor Torque

The torque T developed by the rotor is directly proportional to:

- (i) rotor current
- (ii) rotor e.m.f.
- (iii) power factor of the rotor circuit

 $\therefore$  T  $\propto$  E<sub>2</sub>I<sub>2</sub> cos  $\phi_2$ 

or

 $\mathbf{T} = \mathbf{K} \mathbf{E}_2 \mathbf{I}_2 \cos \phi_2$ 

where

 $I_2 = \text{rotor current at standstill} \\ E_2 = \text{rotor e.m.f. at standstill} \\ \cos \phi_2 = \text{rotor p.f. at standstill} \\$ 

*Note*. The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

# 8.11 Starting Torque (T<sub>s</sub>)

Let  $E_2 = rotor e.m.f.$  per phase at standstill  $X_2 = rotor$  reactance per phase at standstill  $R_2 = rotor$  resistance per phase

Rotor current/phase, 
$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$
 ...at standstill

Rotor p.f., 
$$\cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$
 ...at standstill

$$\therefore$$
 Starting torque,  $T_s = K E_2 I_2 \cos \phi_2$ 

$$= KE_{2} \times \frac{E_{2}}{\sqrt{R_{2}^{2} + X_{2}^{2}}} \times \frac{R_{2}}{\sqrt{R_{2}^{2} + X_{2}^{2}}}$$
$$= \frac{KE_{2}^{2}R_{2}}{R_{2}^{2} + X_{2}^{2}}$$

Generally, the stator supply voltage V is constant so that flux per pole  $\phi$  set up by the stator is also fixed. This in turn means that e.m.f. E<sub>2</sub> induced in the rotor will be constant.

$$\therefore \qquad T_{s} = \frac{K_{1}R_{2}}{R_{2}^{2} + X_{2}^{2}} = \frac{K_{1}R_{2}}{Z_{2}^{2}}$$

where  $K_1$  is another constant.

It is clear that the magnitude of starting torque would depend upon the relative values of  $R_2$  and  $X_2$  i.e., rotor resistance/phase and standstill rotor reactance/phase.

It can be shown that  $K = 3/2 \pi N_s$ .

:. 
$$T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Note that here  $N_s$  is in r.p.s.

#### 8.12 Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

Now 
$$T_s = \frac{K_1 R_2}{R_2^2 + X_2^2}$$
 (i)

Differentiating eq. (i) w.r.t. R<sub>2</sub> and equating the result to zero, we get,

$$\frac{dT_{s}}{dR_{2}} = K_{1} \left[ \frac{1}{R_{2}^{2} + X_{2}^{2}} - \frac{R_{2}(2R_{2})}{\left(R_{2}^{2} + X_{2}^{2}\right)^{2}} \right] = 0$$

or

or  $R_2 = X_2$ 

Hence starting torque will be maximum when:

 $R_2^2 + X_2^2 = 2R_2^2$ 

Rotor resistance/phase = Standstill rotor reactance/phase

Under the condition of maximum starting torque,  $\phi_2 = 45^\circ$  and rotor power factor is 0.707 lagging [See Fig. (8.16 (ii))].

Fig. (8.16 (i)) shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when  $R_2 = X_2$ . If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.



Fig.(8.16)

#### 8.13 Effect of Change of Supply Voltage

$$T_{s} = \frac{KE_{2}^{2}R_{2}}{R_{2}^{2} + X_{2}^{2}}$$

Since  $E_2 \propto$  Supply voltage V

:. 
$$T_s = \frac{K_2 V^2 R_2}{R_2^2 + X_2^2}$$

where  $K_2$  is another constant.

$$\therefore$$
 T<sub>s</sub>  $\propto$  V<sup>2</sup>

Therefore, the starting torque is very sensitive to changes in the value of supply voltage. For example, a drop of 10% in supply voltage will decrease the starting torque by about 20%. This could mean the motor failing to start if it cannot produce a torque greater than the load torque plus friction torque.

#### 8.14 Starting Torque of 3-Phase Induction Motors

The rotor circuit of an induction motor has low resistance and high inductance. At starting, the rotor frequency is equal to the stator frequency (i.e., 50 Hz) so that rotor reactance is large compared with rotor resistance. Therefore, rotor current lags the rotor e.m.f. by a large angle, the power factor is low and consequently the starting torque is small. When resistance is added to the rotor circuit, the rotor power factor is improved which results in improved starting torque. This, of course, increases the rotor impedance and, therefore, decreases the value of rotor current but the effect of improved power factor predominates and the starting torque is increased.

(i) **Squirrel-cage motors**. Since the rotor bars are permanently shortcircuited, it is not possible to add any external resistance in the rotor circuit at starting. Consequently, the stalling torque of such motors is low. Squirrel cage motors have starting torque of 1.5 to 2 times the full-load value with starting current of 5 to 9 times the full-load current.

(ii) Wound rotor motors. The resistance of the rotor circuit of such motors can be increased through the addition of external resistance. By inserting the proper value of external resistance (so that  $R_2 = X_2$ ), maximum starting torque can be obtained. As the motor accelerates, the external resistance is gradually cut out until the rotor circuit is short-circuited on itself for running conditions.

# 8.15 Motor Under Load

Let us now discuss the behaviour of 3-phase induction motor on load.

- (i) When we apply mechanical load to the shaft of the motor, it will begin to slow down and the rotating flux will cut the rotor conductors at a higher and higher rate. The induced voltage and resulting current in rotor conductors will increase progressively, producing greater and greater torque.
- (ii) The motor and mechanical load will soon reach a state of equilibrium when the motor torque is exactly equal to the load torque. When this state is reached, the speed will cease to drop any more and the motor will run at the new speed at a constant rate.
- (iii) The drop in speed of the induction motor on increased load is small. It is because the rotor impedance is low and a small decrease in speed produces a large rotor current. The increased rotor current produces a higher torque to meet the increased load on the motor. This is why induction motors are considered to be constant-speed machines. However, because they never actually turn at synchronous speed, they are sometimes called asynchronous machines.

Note that change in load on the induction motor is met through the adjustment of slip. When load on the motor increases, the slip increases slightly (i.e., motor speed decreases slightly). This results in greater relative speed between the rotating flux and rotor conductors. Consequently, rotor current is increased, producing a higher torque to meet the increased load. Reverse happens should the load on the motor decrease.

(iv) With increasing load, the increased load currents  $I'_2$  are in such a direction so as to decrease the stator flux (Lenz's law), thereby decreasing the counter e.m.f. in the stator windings. The decreased counter e.m.f. allows motor stator current (I<sub>1</sub>) to increase, thereby increasing the power input to the motor. It may be noted that action of the induction motor in adjusting its stator or primary current with

changes of current in the rotor or secondary is very much similar to the changes occurring in transformer with changes in load.



**Fig.(8.17)** 

# **8.16 Torque Under Running Conditions**

Let the rotor at standstill have per phase induced e.m.f. E2, reactance X2 and resistance R<sub>2</sub>. Then under running conditions at slip s,





Running Torque,  $T_r \propto E'_2 I'_2 \cos \phi'_2$  $\propto \phi I'_2 \cos \phi'_2$ (**b** E'<sub>2</sub>  $\propto$   $\phi$ )

$$\propto \phi \times \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (s X_2)^2}} \propto \frac{\phi s E_2 R_2}{R_2^2 + (s X_2)^2} = \frac{K \phi s E_2 R_2}{R_2^2 + (s X_2)^2} = \frac{K_1 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$
 (b E<sub>2</sub> \approx \phi)

If the stator supply voltage V is constant, then stator flux and hence  $E_2$  will be constant.

:. 
$$T_r = \frac{K_2 \ s \ R_2}{R_2^2 + (s \ X_2)^2}$$

where  $K_2$  is another constant.

It may be seen that running torque is:

- (i) directly proportional to slip i.e., if slip increases (i.e., motor speed decreases), the torque will increase and vice-versa.
- (ii) directly proportional to square of supply voltage ( $b E_2 \propto V$ ).

It can be shown that value of  $K_1 = 3/2 \pi N_s$  where  $N_s$  is in r.p.s.

$$\therefore \qquad T_{\rm r} = \frac{3}{2\pi N_{\rm s}} \cdot \frac{{\rm s} \, {\rm E}_2^2 \, {\rm R}_2}{{\rm R}_2^2 + ({\rm s} \, {\rm X}_2)^2} = \frac{3}{2\pi N_{\rm s}} \cdot \frac{{\rm s} \, {\rm E}_2^2 \, {\rm R}_2}{(Z'_2)^2}$$

At starting, s = 1 so that starting torque is

$$T_{s} = \frac{3}{2\pi N_{s}} \cdot \frac{E_{2}^{2} R_{2}}{R_{2}^{2} + X_{2}^{2}}$$

# 8.17 Maximum Torque under Running Conditions

$$T_{\rm r} = \frac{K_2 \ {\rm s} \ {\rm R}_2}{{\rm R}_2^2 + {\rm s}^2 \ {\rm X}_2^2} \tag{i}$$

In order to find the value of rotor resistance that gives maximum torque under running conditions, differentiate exp. (i) w.r.t. s and equate the result to zero i.e.,

$$\frac{dT_{r}}{ds} = \frac{K_{2} \left[ R_{2} \left( R_{2}^{2} + s^{2} X_{2}^{2} \right) - 2s X_{2}^{2} (s R_{2}) \right]}{\left( R_{2}^{2} + s^{2} X_{2}^{2} \right)^{2}} = 0$$

or

$$\left(R_{2}^{2} + s^{2} X_{2}^{2}\right) - 2sX_{2}^{2} = 0$$

 $R_2^2 = s^2 X_2^2$ 

or

or  $R_2 = s X_2$ 

Thus for maximum torque (T<sub>m</sub>) under running conditions :

Rotor resistance/phase = Fractional slip × Standstill rotor reactance/phase

Now  $T_r \propto \frac{s R_2}{R_2^2 + s^2 X_2^2}$  ... from exp. (i) above

For maximum torque,  $R_2 = s X_2$ . Putting  $R_2 = s X_2$  in the above expression, the maximum torque  $T_m$  is given by;

$$T_m \propto \frac{1}{2 X_2}$$

Slip corresponding to maximum torque,  $s = R_2/X_2$ .

It can be shown that:

$$T_{\rm m} = \frac{3}{2\pi N_{\rm s}} \cdot \frac{E_2^2}{2 X_2} \, \rm N - m$$

It is evident from the above equations that:

- (i) The value of rotor resistance does not alter the value of the maximum torque but only the value of the slip at which it occurs.
- (ii) The maximum torque varies inversely as the standstill reactance. Therefore, it should be kept as small as possible.
- (iii) The maximum torque varies directly with the square of the applied voltage.
- (iv) To obtain maximum torque at starting (s = 1), the rotor resistance must be made equal to rotor reactance at standstill.

# 8.18 Torque-Slip Characteristics

As shown in Sec. 8.16, the motor torque under running conditions is given by;

$$T = \frac{K_2 \ s \ R_2}{R_2^2 + s^2 \ X_2^2}$$

If a curve is drawn between the torque and slip for a particular value of rotor resistance  $R_2$ , the graph thus obtained is called torque-slip characteristic. Fig. (8.19) shows a family of torque-slip characteristics for a slip-range from s = 0 to s = 1 for various values of rotor resistance.



**Fig.(8.19)** 

The following points may be noted carefully:

- (i) At s = 0, T = 0 so that torque-slip curve starts from the origin.
- (ii) At normal speed, slip is small so that s  $X_2$  is negligible as compared to  $R_2$ .

$$\therefore \quad T \propto s/R_2$$

$$\propto s \qquad \qquad \dots \text{ as } R_2 \text{ is constant}$$

Hence torque slip curve is a straight line from zero slip to a slip that corresponds to full-load.

(iii) As slip increases beyond full-load slip, the torque increases and becomes maximum at  $s = R_2/X_2$ . This maximum torque in an induction motor is called pull-out torque or break-down torque. Its value is at least twice the full-load value when the motor is operated at rated voltage and frequency.

(iv) to maximum torque, the term 
$$s^2X_2^2$$
 increases very rapidly so that  $R_2^2$  may be neglected as compared to  $s^2X_2^2$ .

$$\therefore \quad \mathbf{T} \propto \mathbf{s}/\mathbf{s}^2 \mathbf{X}_2^2$$
  
\$\infty \left1/\mathbf{s}\$ ... as \$\mathbf{X}\_2\$ is constant

Thus the torque is now inversely proportional to slip. Hence torque-slip curve is a rectangular hyperbola.

(v) The maximum torque remains the same and is independent of the value of rotor resistance. Therefore, the addition of resistance to the rotor circuit does not change the value of maximum torque but it only changes the value of slip at which maximum torque occurs.

# 8.19 Full-Load, Starting and Maximum Torques

$$T_{f} \propto \frac{s R_{2}}{R_{2}^{2} + (s X_{2})^{2}}$$
$$T_{s} \propto \frac{R_{2}}{R_{2}^{2} + X_{2}^{2}}$$
$$T_{m} \propto \frac{1}{2 X_{2}}$$

Note that s corresponds to full-load slip.

(i) 
$$\therefore \frac{T_{m}}{T_{f}} = \frac{R_{2}^{2} + (s X_{2})^{2}}{2s R_{2} X_{2}}$$

Dividing the numerator and denominator on R.H.S. by  $X_2^2$ , we get,

$$\frac{T_{\rm m}}{T_{\rm f}} = \frac{(R_2/X_2)^2 + s^2}{2s(R_2/X_2)} = \frac{a^2 + s^2}{2as}$$

where a =

$$\frac{R_2}{X_2} = \frac{\text{Rotor resistance/phase}}{\text{Standstill rotor reactance/phase}}$$

(ii) 
$$\frac{T_{\rm m}}{T_{\rm s}} = \frac{R_2^2 + X_2^2}{2R_2 X_2}$$

Dividing the numerator and denominator on R.H.S. by  $X_2^2$ , we get,

$$\frac{T_{m}}{T_{f}} = \frac{(R_{2}/X_{2})^{2} + 1}{2(R_{2}/X_{2})} = \frac{a^{2} + 1}{2a}$$
$$a = \frac{R_{2}}{X_{2}} = \frac{\text{Rotor resistance/phase}}{\text{Standstill rotor reactance/phase}}$$

where

# **8.20 Induction Motor and Transformer Compared**

An induction motor may be considered to be a transformer with a rotating shortcircuited secondary. The stator winding corresponds to transformer primary and rotor winding to transformer secondary. However, the following differences between the two are worth noting:

- (i) Unlike a transformer, the magnetic circuit of a 3-phase induction motor has an air gap. Therefore, the magnetizing current in a 3-phase induction motor is much larger than that of the transformer. For example, in an induction motor, it may be as high as 30-50 % of rated current whereas it is only 1-5% of rated current in a transformer.
- (ii) In an induction motor, there is an air gap and the stator and rotor windings are distributed along the periphery of the air gap rather than concentrated

on a core as in a transformer. Therefore, the leakage reactances of stator and rotor windings are quite large compared to that of a transformer.

- (iii) In an induction motor, the inputs to the stator and rotor are electrical but the output from the rotor is mechanical. However, in a transformer, input as well as output is electrical.
- (iv) The main difference between the induction motor and transformer lies in the fact that the rotor voltage and its frequency are both proportional to slip s. If f is the stator frequency,  $E_2$  is the per phase rotor e.m.f. at standstill and  $X_2$  is the standstill rotor reactance/phase, then at any slip s, these values are:

Rotor e.m.f./phase,  $E'_2 = s E_2$ Rotor reactance/phase,  $X'_2 = sX_2$ Rotor frequency, f' = sf

# **8.21 Speed Regulation of Induction Motors**

Like any other electrical motor, the speed regulation of an induction motor is given by:

% age speed regulation =  $\frac{N_0 - N_{F.L.}}{N_{F.L.}} \times 100$ 

where  $N_0$  = no-load speed of the motor  $N_{F.L.}$  = full-load speed of the motor

If the no-load speed of the motor is 800 r.p.m. and its fall-load speed in 780 r.p.m., then change in speed is 800 - 780 = 20 r.p.m. and percentage speed regulation =  $20 \times 100/780 = 2.56\%$ .

At no load, only a small torque is required to overcome the small mechanical losses and hence motor slip is small i.e., about 1%. When the motor is fully loaded, the slip increases slightly i.e., motor speed decreases slightly. It is because rotor impedance is low and a small decrease in speed produces a large rotor current. The increased rotor current produces a high torque to meet the full load on the motor. For this reason, the change in speed of the motor from no-load to full-load is small i.e., the speed regulation of an induction motor is low. The speed regulation of an induction motor is 3% to 5%. Although the motor speed does decrease slightly with increased load, the speed regulation is low enough that the induction motor is classed as a constant-speed motor.

# 8.22 Speed Control of 3-Phase Induction Motors

$$N = (1 - s)N_s = (1 - s)\frac{120 f}{P}$$
(i)

An inspection of eq. (i) reveals that the speed N of an induction motor can be varied by changing (i) supply frequency f (ii) number of poles P on the stator and (iii) slip s. The change of frequency is generally not possible because the commercial supplies have constant frequency. Therefore, the practical methods of speed control are either to change the number of stator poles or the motor slip.

#### 1. Squirrel cage motors

The speed of a squirrel cage motor is changed by changing the number of stator poles. Only two or four speeds are possible by this method. Two-speed motor has one stator winding that may be switched through suitable control equipment to provide two speeds, one of which is half of the other. For instance, the winding may be connected for either 4 or 8 poles, giving synchronous speeds of 1500 and 750 r.p.m. Four-speed motors are equipped with two separate stator windings each of which provides two speeds. The disadvantages of this method are:

- (i) It is not possible to obtain gradual continuous speed control.
- (ii) Because of the complications in the design and switching of the interconnections of the stator winding, this method can provide a maximum of four different synchronous speeds for any one motor.

#### 2. Wound rotor motors

The speed of wound rotor motors is changed by changing the motor slip. This can be achieved by;

- (i) varying the stator line voltage
- (ii) varying the resistance of the rotor circuit
- (iii) inserting and varying a foreign voltage in the rotor circuit

# **8.23 Power Factor of Induction Motor**

Like any other a.c. machine, the power factor of an induction motor is given by;

Power factor, 
$$\cos \phi = \frac{\text{Active component of current } (I \cos \phi)}{\text{Total current } (I)}$$

The presence of air-gap between the stator and rotor of an induction motor greatly increases the reluctance of the magnetic circuit. Consequently, an induction motor draws a large magnetizing current  $(I_m)$  to produce the required flux in the air-gap.

(i) At no load, an induction motor draws a large magnetizing current and a small active component to meet the no-load losses. Therefore, the induction motor takes a high no-load current lagging the applied voltage

by a large angle. Hence the power factor of an induction motor on no load is low i.e., about 0.1 lagging.

(ii) When an induction motor is loaded, the active component of current increases while the magnetizing component remains about the same. Consequently, the power factor of the motor is increased. However, because of the large value of magnetizing current, which is present regardless of load, the power factor of an induction motor even at full-load seldom exceeds 0.9 lagging.

# **8.24** Power Stages in an Induction Motor

The input electric power fed to the stator of the motor is converted into mechanical power at the shaft of the motor. The various losses during the energy conversion are:

#### 1. Fixed losses

- (i) Stator iron loss
- (ii) Friction and windage loss

The rotor iron loss is negligible because the frequency of rotor currents under normal running condition is small.

#### 2. Variable losses

(i)

- (i) Stator copper loss
- (ii) Rotor copper loss

Fig. (8.20) shows how electric power fed to the stator of an induction motor suffers losses and finally converted into mechanical power.

The following points may be noted from the above diagram:

Stator input,  $P_i$  = Stator output + Stator losses

= Stator output + Stator Iron loss + Stator Cu loss

- (ii) Rotor input,  $P_r$  = Stator output It is because stator output is entirely transferred to the rotor through airgap by electromagnetic induction.
- (iii) Mechanical power available,  $P_m = P_r Rotor Cu loss$ This mechanical power available is the gross rotor output and will produce a gross torque  $T_g$ .
- (iv) Mechanical power at shaft,  $P_{out} = P_m Friction$  and windage loss Mechanical power available at the shaft produces a shaft torque  $T_{sh}$ .

Clearly,  $P_m - P_{out} =$  Friction and windage loss



#### **8.25 Induction Motor Torque**

The mechanical power P available from any electric motor can be expressed as:

$$P = \frac{2\pi NT}{60} \quad \text{watts}$$

where

N = speed of the motor in r.p.m. T = torque developed in N-m

:. 
$$T = \frac{60}{2\pi} \frac{P}{N} = 9.55 \frac{P}{N} N - m$$

If the gross output of the rotor of an induction motor is  $P_m$  and its speed is N r.p.m., then gross torque T developed is given by:

$$T_g = 9.55 \frac{P_m}{N} N - m$$

 $T_{sh} = 9.55 \frac{P_{out}}{N} N - m$ 

Similarly,

*Note*. Since windage and friction loss is small,  $T_g = T_{sh}$ . This assumption hardly leads to any significant error.

#### 8.26 Rotor Output

If  $T_g$  newton-metre is the gross torque developed and N r.p.m. is the speed of the rotor, then,

Gross rotor output = 
$$\frac{2\pi N T_g}{60}$$
 watts

If there were no copper losses in the rotor, the output would equal rotor input and the rotor would run at synchronous speed  $N_s$ .

 Rotor input =	$\frac{2\pi N_s T_g}{60}$	watts
	00	

 $\therefore$  Rotor Cu loss = Rotor input – Rotor output

$$=\frac{2\pi T_g}{60}(N_s-N)$$

- (i)  $\frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{N_s N}{N_s} = s$ 
  - $\therefore$  Rotor Cu loss = s × Rotor input

(ii) Gross rotor output, 
$$P_m = Rotor input - Rotor Cu loss$$
  
= Rotor input - s × Rotor input  
 $\therefore$   $P_m = Rotor input (1 - s)$ 

(iii) 
$$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s = \frac{N}{N_s}$$

(iv) 
$$\frac{\text{Rotor Culloss}}{\text{Gross rotor output}} = \frac{s}{1-s}$$

It is clear that if the input power to rotor is  $P_r$  then s  $P_r$  is lost as rotor Cu loss and the remaining  $(1 - s)P_r$  is converted into mechanical power. Consequently, induction motor operating at high slip has poor efficiency.

Note.

$$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s$$

If the stator losses as well as friction and windage losses arc neglected, then,

Gross rotor output = Useful output

Rotor input = Stator input

$$\therefore \frac{\text{Useful output}}{\text{Stator output}} = 1 - s = \text{Efficiency}$$

Hence the approximate efficiency of an induction motor is 1 - s. Thus if the slip of an induction motor is 0.125, then its approximate efficiency is = 1 - 0.125 = 0.875 or 87.5%.

# **8.27 Induction Motor Torque Equation**

The gross torque T<sub>g</sub> developed by an induction motor is given by;

$$T_{g} = \frac{\text{Rotor input}}{2\pi N_{s}} \qquad \dots N_{s} \text{ is r.p.s.}$$
$$= \frac{60 \times \text{Rotor input}}{2\pi N_{s}} \qquad (\text{See Sec. 8.26}) \qquad \dots N_{s} \text{ is r.p.s.}$$

Now Rotor input = 
$$\frac{\text{Rotor Cu loss}}{s} = \frac{3(\Gamma_2)^2 R_2}{s}$$
 (i)

As shown in Sec. 8.16, under running conditions,

$$I'_{2} = \frac{s E_{2}}{\sqrt{R_{2}^{2} + (s X_{2})^{2}}} = \frac{s K E_{1}}{\sqrt{R_{2}^{2} + (s X_{2})^{2}}}$$

where  $K = Transformation ratio = \frac{Rotor turns/phase}{Stator turns/phase}$ 

$$\therefore \text{ Rotor input} = 3 \times \frac{s^2 E_2^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{1}{s} = \frac{3 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

(Putting me value of I'<sub>2</sub> in eq.(i))

Also Rotor input = 
$$3 \times \frac{s^2 K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{1}{s} = \frac{3 s K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2}$$

(Putting me value of I'<sub>2</sub> in eq.(i))

$$\therefore \quad T_{g} = \frac{\text{Rotor input}}{2\pi N_{s}} = \frac{3}{2\pi N_{s}} \times \frac{s E_{2}^{2} R_{2}}{R_{2}^{2} + (s X_{2})^{2}} \qquad \text{...in terms of } E_{2}$$
$$= \frac{3}{2\pi N_{s}} \times \frac{s K^{2} E_{1}^{2} R_{2}}{R_{2}^{2} + (s X_{2})^{2}} \qquad \text{...in terms of } E_{1}$$

Note that in the above expressions of  $T_g$ , the values  $E_1$ ,  $E_2$ ,  $R_2$  and  $X_2$  represent the phase values.

#### 8.28 Performance Curves of Squirrel-Cage Motor

The performance curves of a 3-phase induction motor indicate the variations of speed, power factor, efficiency, stator current and torque for different values of load. However, before giving the performance curves in one graph, it is desirable to discuss the variation of torque, and stator current with slip.

#### (i) Variation of torque and stator current with slip

Fig. (8.21) shows the variation of torque and stator current with slip for a standard squirrel-cage motor. Generally, the rotor resistance is low so that full-

load current occurs at low slip. Then even at full-load f' (= sf) and. therefore,  $X'_2$  $(= 2\pi f' L_2)$  are low. Between zero and full-load, rotor power factor  $(= \cos \phi'_2)$ and rotor impedance  $(= Z'_2)$  remain practically constant. Therefore, rotor current  $I'_2(E'_2/Z'_2)$  and, therefore, torque  $(T_r)$  increase directly with the slip. Now stator current  $I_1$  increases in proportion to  $I'_2$ . This is shown in Fig. (8.21) where  $T_r$  and  ${\rm I}_1$  are indicated as straight lines from no-load to full-load. As load and slip are increased beyond full-load, the increase in rotor reactance becomes appreciable. The increasing value of rotor impedance not only decreases the rotor power factor  $\cos \phi'_2$  (=  $R_2/Z'_2$ ) but also lowers the rate of increase of rotor current. As a result, the torque  $T_r$  and stator current  $I_1$  do not increase directly with slip as indicated in Fig. (8.21). With the decreasing power factor and the lowered rate of increase in rotor current, the stator current  $I_1$  and torque  $T_r$  increase at a lower rate. Finally, torque  $T_r$  reaches the maximum value at about 25% slip in the standard squirrel cage motor. This maximum value of torque is called the pullout torque or breakdown torque. If the load is increased beyond the breakdown point, the decrease in rotor power factor is greater than the increase in rotor current, resulting in a decreasing torque. The result is that motor slows down quickly and comes to a stop.



In Fig. (8.21), the value of torque at starting (i.e., s = 100%) is 1.5 times the full-load torque. The starting current is about five times the full-load current. The motor is essentially a constant-speed machine having speed characteristics about the same as a d.c. shunt motor.

#### (ii) Performance curves

Fig. (8.22) shows the performance curves of 3-phase squirrel cage induction motor.



**Fig.**(8.22)

The following points may be noted:

- (a) At no-load, the rotor lags behind the stator flux by only a small amount, since the only torque required is that needed to overcome the no-load losses. As mechanical load is added, the rotor speed decreases. A decrease in rotor speed allows the constant-speed rotating field to sweep across the rotor conductors at a faster rate, thereby inducing large rotor currents. This results in a larger torque output at a slightly reduced speed. This explains for speed-load curve in Fig. (8.22).
- (b) At no-load, the current drawn by an induction motor is largely a magnetizing current; the no-load current lagging the applied voltage by a large angle. Thus the power factor of a lightly loaded induction motor is very low. Because of the air gap, the reluctance of the magnetic circuit is high, resulting in a large value of no-load current as compared with a transformer. As load is added, the active or power component of current increases, resulting in a higher power factor. However, because of the large value of magnetizing current, which is present regardless of load, the power factor of an induction motor even at full-load seldom exceeds 90%. Fig. (8.22) shows the variation of power factor with load of a typical squirrel-cage induction motor.
- (c) Efficiency =  $\frac{\text{Output}}{\text{Output} + \text{Losses}}$

The losses occurring in a 3-phase induction motor are Cu losses in stator and rotor windings, iron losses in stator and rotor core and friction and windage losses. The iron losses and friction and windage losses are almost independent of load. Had  $I^2R$  been constant, the efficiency of the motor would have increased with load. But  $I^2R$  loss depends upon load. Therefore, the efficiency of the motor increases with load but the curve is dropping at high loads.

- (d) At no-load, the only torque required is that needed to overcome no-load losses. Therefore, stator draws a small current from the supply. As mechanical load is added, the rotor speed decreases. A decrease in rotor speed allows the constant-speed rotating field to sweep across the rotor conductors at a faster rate, thereby inducing larger rotor currents. With increasing loads, the increased rotor currents are in such a direction so as to decrease the staler flux, thereby temporarily decreasing the counter e.m.f. in the stator winding. The decreased counter e.m.f. allows more stator current to flow.
- (e) Output = Torque × Speed Since the speed of the motor does not change appreciably with load, the torque increases with increase in load.

# 8.29 Equivalent Circuit of 3-Phase Induction Motor at Any Slip

In a 3-phase induction motor, the stator winding is connected to 3-phase supply and the rotor winding is short-circuited. The energy is transferred magnetically from the stator winding to the short-circuited, rotor winding. Therefore, an induction motor may be considered to be a transformer with a rotating secondary (short-circuited). The stator winding corresponds to transformer primary and the rotor finding corresponds to transformer secondary. In view of the similarity of the flux and voltage conditions to those in a transformer, one can expect that the equivalent circuit of an induction motor will be similar to that of a transformer. Fig. (8.23) shows the equivalent circuit (though not the only one) per phase for an induction motor. Let us discuss the stator and rotor circuits separately.



**Stator circuit**. In the stator, the events are very similar to those in the transformer primal y. The applied voltage per phase to the stator is  $V_1$  and  $R_1$  and  $X_1$  are the stator resistance and leakage reactance per phase respectively. The applied voltage  $V_1$  produces a magnetic flux which links the stator winding (i.e., primary) as well as the rotor winding (i.e., secondary). As a result, self-

induced e.m.f.  $E_1$  is induced in the stator winding and mutually induced e.m.f.  $E'_2$ (= s  $E_2$  = s K  $E_1$  where K is transformation ratio) is induced in the rotor winding. The flow of stator current I<sub>1</sub> causes voltage drops in R<sub>1</sub> and X<sub>1</sub>.

:. 
$$V_1 = -E_1 + I_1(R_1 + j X_1)$$
 ...phasor sum

When the motor is at no-load, the stator winding draws a current  $I_0$ . It has two components viz., (i) which supplies the no-load motor losses and (ii) magnetizing component  $I_m$  which sets up magnetic flux in the core and the airgap. The parallel combination of  $R_c$  and  $X_m$ , therefore, represents the no-load motor losses and the production of magnetic flux respectively.

$$\mathbf{I}_0 = \mathbf{I}_{w} + \mathbf{I}_{m}$$

.

**Rotor circuit**. Here  $R_2$  and  $X_2$  represent the rotor resistance and standstill rotor reactance per phase respectively. At any slip s, the rotor reactance will be s  $X_2$  The induced voltage/phase in the rotor is  $E'_2 = s E_2 = s K E_1$ . Since the rotor winding is short-circuited, the whole of e.m.f.  $E'_2$  is used up in circulating the rotor current  $I'_2$ .

$$E'_2 = I'_2 (R_2 + j s X_2)$$

The rotor current  $I'_2$  is reflected as  $I''_2$  (= K  $I'_2$ ) in the stator. The phasor sum of  $I''_2$  and  $I_0$  gives the stator current  $I_1$ .

It is important to note that input to the primary and output from the secondary of a transformer are electrical. However, in an induction motor, the inputs to the stator and rotor are electrical but the output from the rotor is mechanical. To

facilitate calculations, it is desirable and necessary to replace the mechanical load by an equivalent electrical load. We then have the transformer equivalent circuit of the induction motor.

It may be noted that even though the frequencies of stator and rotor currents are different, yet the magnetic fields due to them rotate at synchronous speed  $N_s$ . The stator currents produce a magnetic flux which rotates at a speed  $N_s$ . At slip s, the speed of rotation of the rotor field relative to the rotor surface in the direction of rotation of the rotor is

$$=\frac{120 \text{ f'}}{\text{P}}=\frac{120 \text{ s f}}{\text{P}}=\text{s N}_{\text{s}}$$

But the rotor is revolving at a speed of N relative to the stator core. Therefore, the speed of rotor



**Fig.**(8.24)

field relative to stator core

 $= sN_s + N = (N_s - N) + N = N_s$ 

Thus no matter what the value of slip s, the stator and rotor magnetic fields are synchronous with each other when seen by an observer stationed in space. Consequently, the 3-phase induction motor can be regarded as being equivalent to a transformer having an air-gap separating the iron portions of the magnetic circuit carrying the primary and secondary windings.

Fig. (8.24) shows the phasor diagram of induction motor.

#### 8.30 Equivalent Circuit of the Rotor

We shall now see how mechanical load of the motor is replaced by the equivalent electrical load. Fig. (8.25 (i)) shows the equivalent circuit per phase of the rotor at slip s. The rotor phase current is given by;

$$I'_{2} = \frac{s E_{2}}{\sqrt{R_{2}^{2} + (s X_{2})^{2}}}$$

Mathematically, this value is unaltered by writing it as:

$$I'_{2} = \frac{E_{2}}{\sqrt{(R_{2}/s)^{2} + (X_{2})^{2}}}$$

As shown in Fig. (8.25 (ii)), we now have a rotor circuit that has a fixed reactance  $X_2$  connected in series with a variable resistance  $R_2$ /s and supplied with constant voltage  $E_2$ . Note that Fig. (8.25 (ii)) transfers the variable to the resistance without altering power or power factor conditions.



The quantity  $R_2/s$  is greater than  $R_2$  since s is a fraction. Therefore,  $R_2/s$  can be divided into a fixed part  $R_2$  and a variable part  $(R_2/s - R_2)$  i.e.,

$$\frac{\mathbf{R}_2}{\mathbf{s}} = \mathbf{R}_2 + \mathbf{R}_2 \left(\frac{1}{\mathbf{s}} - 1\right)$$

- (i) The first part  $R_2$  is the rotor resistance/phase, and represents the rotor Cu loss.
- (ii) The second part  $R_2\left(\frac{1}{s}-1\right)$  is a variable-resistance load. The power delivered to this load represents the total mechanical power developed in the rotor. Thus mechanical load on the induction motor can be replaced by a variable-resistance load of value  $R_2\left(\frac{1}{s}-1\right)$ . This is

$$\therefore \qquad \mathbf{R}_{\mathrm{L}} = \mathbf{R}_{2} \left( \frac{1}{s} - 1 \right)$$

Fig. (8.25 (iii)) shows the equivalent rotor circuit along with load resistance  $R_L$ .

#### 8.31 Transformer Equivalent Circuit of Induction Motor

Fig. (8.26) shows the equivalent circuit per phase of a 3-phase induction motor. Note that mechanical load on the motor has been replaced by an equivalent electrical resistance  $R_L$  given by;



Note that circuit shown in Fig. (8.26) is similar to the equivalent circuit of a transformer with secondary load equal to  $R_2$  given by eq. (i). The rotor e.m.f. in the equivalent circuit now depends only on the transformation ratio K (=  $E_2/E_1$ ).

Therefore; induction motor can be represented as an equivalent transformer connected to a variable-resistance load  $R_L$  given by eq. (i). The power delivered to  $R_L$  represents the total mechanical power developed in the rotor. Since the equivalent circuit of Fig. (8.26) is that of a transformer, the secondary (i.e., rotor) values can be transferred to primary (i.e., stator) through the appropriate use of transformation ratio K. Recall that when shifting resistance/reactance from secondary to primary, it should be divided by  $K^2$  whereas current should be multiplied by K. The equivalent circuit of an induction motor referred to primary is shown in Fig. (8.27).



**Fig.**(8.27)

Note that the element (i.e.,  $R'_L$ ) enclosed in the dotted box is the equivalent electrical resistance related to the mechanical load on the motor. The following points may be noted from the equivalent circuit of the induction motor:

- (i) At no-load, the slip is practically zero and the load  $R'_L$  is infinite. This condition resembles that in a transformer whose secondary winding is open-circuited.
- (ii) At standstill, the slip is unity and the load  $R'_L$  is zero. This condition resembles that in a transformer whose secondary winding is short-circuited.
- (iii) When the motor is running under load, the value of  $R'_L$  will depend upon the value of the slip s. This condition resembles that in a transformer whose secondary is supplying variable and purely resistive load.
- (iv) The equivalent electrical resistance  $R'_L$  related to mechanical load is slip or speed dependent. If the slip s increases, the load  $R'_L$  decreases and the rotor current increases and motor will develop more mechanical power. This is expected because the slip of the motor increases with the increase of load on the motor shaft.

# 8.32 Power Relations

The transformer equivalent circuit of an induction motor is quite helpful in analyzing the various power relations in the motor. Fig. (8.28) shows the equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).



(i) Total electrical load =  $R'_2\left(\frac{1}{s}-1\right) + R'_2 = \frac{R'_2}{s}$ Power input to stator =  $3V_1 I_1 \cos \phi_1$ 

There will be stator core loss and stator Cu loss. The remaining power will be the power transferred across the air-gap i.e., input to the rotor.

(ii) Rotor input = 
$$\frac{3(I''_2)^2 R'_2}{s}$$
  
Rotor Cu loss =  $3(I''_2)^2 R'_2$ 

Total mechanical power developed by the rotor is

$$P_{\rm m} = \text{Rotor input} - \text{Rotor Cu loss}$$
$$= \frac{3(I''_2)^2 R'_2}{s} 3(I''_2)^2 R'_2 = 3(I''_2)^2 R'_2 \left(\frac{1}{s} - 1\right)$$

This is quite apparent from the equivalent circuit shown in Fig. (8.28).

(iii) If  $T_g$  is the gross torque developed by the rotor, then,

$$P_{m} = \frac{2\pi N T_{g}}{60}$$
  
or  $3(I'_{2})^{2} R'_{2} \left(\frac{1}{s} - 1\right) = \frac{2\pi N T_{g}}{60}$   
or  $3(I'_{2})^{2} R'_{2} \left(\frac{1 - s}{s}\right) = \frac{2\pi N T_{g}}{60}$   
or  $3(I'_{2})^{2} R'_{2} \left(\frac{1 - s}{s}\right) = \frac{2\pi N_{s}(1 - s) T_{g}}{60}$  [b N = N<sub>s</sub>(1 - s)]  
 $\therefore T_{g} = \frac{3(I'_{2})^{2} R'_{2}/s}{2\pi N_{s}/60}$  N - m  
or  $T_{g} = 9.55 \frac{3(I'_{2})^{2} R'_{2}/s}{N_{s}}$  N - m

Note that shaft torque  $T_{sh}$  will be less than  $T_g$  by the torque required to meet windage and frictional losses.

# 8.33 Approximate Equivalent Circuit of Induction Motor

As in case of a transformer, the approximate equivalent circuit of an induction motor is obtained by shifting the shunt branch  $(R_c - X_m)$  to the input terminals as shown in Fig. (8.29). This step has been taken on the assumption that voltage drop in  $R_1$  and  $X_1$  is small and the terminal voltage  $V_1$  does not appreciably differ from the induced voltage  $E_1$ . Fig. (8.29) shows the approximate equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).



Fig.(8.29)

The above approximate circuit of induction motor is not so readily justified as with the transformer. This is due to the following reasons:

- (i) Unlike that of a power transformer, the magnetic circuit of the induction motor has an air-gap. Therefore, the exciting current of induction motor (30 to 40% of full-load current) is much higher than that of the power transformer. Consequently, the exact equivalent circuit must be used for accurate results.
- (ii) The relative values of  $X_1$  and  $X_2$  in an induction motor are larger than the corresponding ones to be found in the transformer. This fact does not justify the use of approximate equivalent circuit
- (iii) In a transformer, the windings are concentrated whereas in an induction motor, the windings are distributed. This affects the transformation ratio.

In spite of the above drawbacks of approximate equivalent circuit, it yields results that are satisfactory for large motors. However, approximate equivalent circuit is not justified for small motors.

# 8.34 Starting of 3-Phase Induction Motors

The induction motor is fundamentally a transformer in which the stator is the primary and the rotor is short-circuited secondary. At starting, the voltage induced in the induction motor rotor is maximum ( $\mathbf{b} \ s = 1$ ). Since the rotor impedance is low, the rotor current is excessively large. This large rotor current is reflected in the stator because of transformer action. This results in high starting current (4 to 10 times the full-load current) in the stator at low power
factor and consequently the value of starting torque is low. Because of the short duration, this value of large current does not harm the motor if the motor accelerates normally. However, this large starting current will produce large line-voltage drop. This will adversely affect the operation of other electrical equipment connected to the same lines. Therefore, it is desirable and necessary to reduce the magnitude of stator current at starting and several methods are available for this purpose.

# **8.35 Methods of Starting 3-Phase Induction Motors**

The method to be employed in starting a given induction motor depends upon the size of the motor and the type of the motor. The common methods used to start induction motors are:

- (i) Direct-on-line starting
- (ii) Stator resistance starting
- (iii) Autotransformer starting
- (iv) Star-delta starting
- (v) Rotor resistance starting

Methods (i) to (iv) are applicable to both squirrel-cage and slip ring motors. However, method (v) is applicable only to slip ring motors. In practice, any one of the first four methods is used for starting squirrel cage motors, depending upon the size of the motor. But slip ring motors are invariably started by rotor resistance starting.

# **8.36 Methods of Starting Squirrel-Cage Motors**

Except direct-on-line starting, all other methods of starting squirrel-cage motors employ reduced voltage across motor terminals at starting.

## (i) Direct-on-line starting

This method of starting in just what the name implies—the motor is started by connecting it directly to 3-phase supply. The impedance of the motor at standstill is relatively low and when it is directly connected to the supply system, the starting current will be high (4 to 10 times the full-load current) and at a low power factor. Consequently, this method of starting is suitable for relatively small (up to 7.5 kW) machines.

Relation between starling and F.L. torques. We know that:

Rotor input =  $2\pi N_s T = kT$ 

But Rotor Cu loss =  $s \times Rotor$  input

 $\therefore \quad 3(\mathbf{I'}_2)^2 \mathbf{R}_2 = \mathbf{s} \times \mathbf{k} \mathbf{T}$ 

or

 $T \propto (I'_2)^2/s$ 

$$T \propto I_1^2/s$$
 (b  $I'_2 \propto I_1$ )

If  $I_{st}$  is the starting current, then starting torque ( $T_{st}$ ) is

$$\propto I_{st}^2$$
 (b at starting s = 1)

If I<sub>f</sub> is the full-load current and s<sub>f</sub> is the full-load slip, then,

$$T_{f} \propto I_{f}^{2} / s_{f}$$
$$\therefore \qquad \frac{T_{st}}{T_{f}} = \left(\frac{I_{st}}{I_{f}}\right)^{2} \times s_{f}$$

Т

When the motor is started direct-on-line, the starting current is the short-circuit (blocked-rotor) current  $I_{sc}$ .

$$\therefore \qquad \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

Let us illustrate the above relation with a numerical example. Suppose  $I_{sc} = 5 I_f$  and full-load slip  $s_f$  =0.04. Then,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f = \left(\frac{5 I_f}{I_f}\right)^2 \times 0.04 = (5)^2 \times 0.04 = 1$$
$$T_{st} = T_f$$

Note that starting current is as large as five times the full-load current but starting torque is just equal to the full-load torque. Therefore, starting current is very high and the starting torque is comparatively low. If this large starting current flows for a long time, it may overheat the motor and damage the insulation.

#### (ii) Stator resistance starting

...

In this method, external resistances are connected in series with each phase of stator winding during starting. This causes voltage drop across the resistances so that voltage available across motor terminals is reduced and hence the starting current. The starting resistances are gradually cut out in steps (two or more steps) from the stator circuit as the motor picks up speed. When the motor attains rated speed, the resistances are completely cut out and full line voltage is applied to the rotor.

This method suffers from two drawbacks. First, the reduced voltage applied to the motor during the starting period lowers the starting torque and hence increases the accelerating time. Secondly, a lot of power is wasted in the starting resistances.

or



**Relation between starting and F.L. torques**. Let V be the rated voltage/phase. If the voltage is reduced by a fraction x by the insertion of resistors in the line, then voltage applied to the motor per phase will be xV.

Now

or

 $\frac{T_{st}}{T_{f}} = \left(\frac{I_{st}}{I_{f}}\right)^{2} \times s_{f}$  $\frac{T_{st}}{T_{f}} = x^{2} \left(\frac{I_{sc}}{I_{f}}\right)^{2} \times s_{f}$ 

 $I_{st} = X I_{sc}$ 

Thus while the starting current reduces by a fraction x of the rated-voltage starting current ( $I_{sc}$ ), the starting torque is reduced by a fraction  $x^2$  of that obtained by direct switching. The reduced voltage applied to the motor during the starting period lowers the starting current but at the same time increases the accelerating time because of the reduced value of the starting torque. Therefore, this method is used for starting small motors only.

#### (iii) Autotransformer starting

This method also aims at connecting the induction motor to a reduced supply at starting and then connecting it to the full voltage as the motor picks up sufficient speed. Fig. (8.31) shows the circuit arrangement for autotransformer starting. The tapping on the autotransformer is so set that when it is in the circuit, 65% to 80% of line voltage is applied to the motor.

At the instant of starting, the change-over switch is thrown to "start" position. This puts the autotransformer in the circuit and thus reduced voltage is applied to the circuit. Consequently, starting current is limited to safe value. When the motor attains about 80% of normal speed, the changeover switch is thrown to

"run" position. This takes out the autotransformer from the circuit and puts the motor to full line voltage. Autotransformer starting has several advantages viz low power loss, low starting current and less radiated heat. For large machines (over 25 H.P.), this method of starting is often used. This method can be used for both star and delta connected motors.



Fig.(8.31)

**Relation between starting And F.L. torques**. Consider a star-connected squirrel-cage induction motor. If V is the line voltage, then voltage across motor phase on direct switching is  $V/\sqrt{3}$  and starting current is  $I_{st} = I_{sc}$ . In case of autotransformer, if a tapping of transformation ratio K (a fraction) is used, then phase voltage across motor is  $KV/\sqrt{3}$  and  $I_{st} = K I_{sc}$ ,

Now



**Fig.(8.32)** 

The current taken from the supply or by autotransformer is  $I_1 = KI_2 = K^2 I_{sc}$ . Note that motor current is K times, the supply line current is  $K^2$  times and the starting torque is  $K^2$  times the value it would have been on direct-on-line starting.

#### (iv) Star-delta starting

The stator winding of the motor is designed for delta operation and is connected in star during the starting period. When the machine is up to speed, the connections are changed to delta. The circuit arrangement for star-delta starting is shown in Fig. (8.33).

The six leads of the stator windings are connected to the changeover switch as shown. At the instant of starting, the changeover switch is thrown to "Start" position which connects the stator windings in star. Therefore, each stator phase gets  $V/\sqrt{3}$  volts where V is the line voltage. This reduces the starting current. When the motor picks up speed, the changeover switch is thrown to "Run" position which connects the stator windings in delta. Now each stator phase gets full line voltage V. The disadvantages of this method are:

- (a) With star-connection during starting, stator phase voltage is  $1/\sqrt{3}$  times the line voltage. Consequently, starting torque is  $(1/\sqrt{3})^2$  or 1/3 times the value it would have with  $\Delta$ -connection. This is rather a large reduction in starting torque.
- (b) The reduction in voltage is fixed.

This method of starting is used for medium-size machines (upto about 25 H.P.).

#### Relation between starting and F.L. torques. In direct delta starting,

Starting current/phase,  $I_{sc} = V/Z_{sc}$  where V = line voltage

Starting line current =  $\sqrt{3}$  I<sub>sc</sub>

In star starting, we have,

Starting current/phase, 
$$I_{st} = \frac{V/\sqrt{3}}{Z_{sc}} = \frac{1}{\sqrt{3}}I_{sc}$$

Now

$$\frac{T_{st}}{T_{f}} = \left(\frac{I_{st}}{I_{f}}\right)^{2} \times s_{f} = \left(\frac{I_{sc}}{\sqrt{3} \times I_{f}}\right)^{2} \times s_{f}$$

or

where 
$$I_{sc} = \text{starting phase current (delta)}$$
  
 $I_f = F.L.$  phase current (delta)

 $\frac{T_{st}}{T_{s}} = \frac{1}{3} \left(\frac{I_{sc}}{I}\right)^2 \times s_f$ 



**Fig.(8.33)** 

Note that in star-delta starting, the starting line current is reduced to one-third as compared to starting with the winding delta connected. Further, starting torque is reduced to one-third of that obtainable by direct delta starting. This method is cheap but limited to applications where high starting torque is not necessary e.g., machine tools, pumps etc.

## 8.37 Starting of Slip-Ring Motors

Slip-ring motors are invariably started by rotor resistance starting. In this method, a variable star-connected rheostat is connected in the rotor circuit through slip rings and full voltage is applied to the stator winding as shown in Fig. (8.34).



Fig.(8.34)

- (i) At starting, the handle of rheostat is set in the OFF position so that maximum resistance is placed in each phase of the rotor circuit. This reduces the starting current and at the same time starting torque is increased.
- (ii) As the motor picks up speed, the handle of rheostat is gradually moved in clockwise direction and cuts out the external resistance in each phase of the rotor circuit. When the motor attains normal speed, the change-over switch is in the ON position and the whole external resistance is cut out from the rotor circuit.

# 8.38 Slip-Ring Motors Versus Squirrel Cage Motors

The slip-ring induction motors have the following advantages over the squirrel cage motors:

- (i) High starting torque with low starting current.
- (ii) Smooth acceleration under heavy loads.
- (iii) No abnormal heating during starting.
- (iv) Good running characteristics after external rotor resistances are cut out.
- (v) Adjustable speed.

The disadvantages of slip-ring motors are:

- (i) The initial and maintenance costs are greater than those of squirrel cage motors.
- (ii) The speed regulation is poor when run with resistance in the rotor circuit

# 8.39 Induction Motor Rating

The nameplate of a 3-phase induction motor provides the following information:

- (i) Horsepower (ii) Line voltage (iii) Line current
- (iv) Speed (v) Frequency (vi) Temperature rise

The horsepower rating is the mechanical output of the motor when it is operated at rated line voltage, rated frequency and rated speed. Under these conditions, the line current is that specified on the nameplate and the temperature rise does not exceed that specified.

The speed given on the nameplate is the actual speed of the motor at rated fullload; it is not the synchronous speed. Thus, the nameplate speed of the induction motor might be 1710 r.p.m. It is the rated full-load speed.

# 8.40 Double Squirrel-Cage Motors

One of the advantages of the slip-ring motor is that resistance may be inserted in the rotor circuit to obtain high starting torque (at low starting current) and then cut out to obtain optimum running conditions. However, such a procedure cannot be adopted for a squirrel cage motor because its cage is permanently short-circuited. In order to provide high starting torque at low starting current, double-cage construction is used.

## Construction

As the name suggests, the rotor of this motor has two squirrel-cage windings located one above the other as shown in Fig. (8.35 (i)).

(i) **The outer winding** consists of bars of smaller cross-section short-circuited by end rings. Therefore, the resistance of this winding is high. Since the

outer winding has relatively open slots and a poorer flux path around its bars [See Fig. (8.35 (ii))], it has a low inductance. Thus the resistance of the outer squirrel-cage winding is high and its inductance is low.

(ii) **The inner winding** consists of bars of greater cross-section short-circuited by end rings. Therefore, the resistance of this winding is low. Since the bars of the inner winding are thoroughly buried in iron, it has a high inductance [See Fig. (8.35 (ii))]. Thus the resistance of the inner squirrelcage winding is low and its inductance is high.



#### Working

When a rotating magnetic field sweeps across the two windings, equal e.m.f.s are induced in each.

- (i) At starting, the rotor frequency is the same as that of the line (i.e., 50 Hz), making the reactance of the lower winding much higher than that of the upper winding. Because of the high reactance of the lower winding, nearly all the rotor current flows in the high-resistance outer cage winding. This provides the good starting characteristics of a high-resistance cage winding. Thus the outer winding gives high starting torque at low starting current.
- (ii) As the motor accelerates, the rotor frequency decreases, thereby lowering the reactance of the inner winding, allowing it to carry a larger proportion of the total rotor current At the normal operating speed of the motor, the rotor frequency is so low (2 to 3 Hz) that nearly all the rotor current flows in the low-resistance inner cage winding. This results in good operating efficiency and speed regulation.

Fig. (8.36) shows the operating characteristics of double squirrel-cage motor. The starting torque of this motor ranges from 200 to 250 percent of full-load torque with a starting current of 4 to 6 times the full-load value. It is classed as a high-torque, low starting current motor.



## 8.41 Equivalent Circuit of Double Squirrel-Cage Motor

Fig. (8.37) shows a section of the double squirrel cage motor. Here  $R_o$  and  $R_i$  are the per phase resistances of the outer cage winding and inner cage winding whereas  $X_o$  and  $X_i$  are the corresponding per phase standstill reactances. For the outer cage, the resistance is made intentionally high, giving a high starting torque. For the inner cage winding, the resistance is low and the leakage reactance is high, giving a low starting torque



but high efficiency on load. Note that in a double squirrel cage motor, the outer winding produces the high starting and accelerating torque while the inner winding provides the running torque at good efficiency.

Fig. (8.38 (i)) shows the equivalent circuit for one phase of double cage motor referred to stator. The two cage impedances are effectively in parallel. The resistances and reactances of the outer and inner rotors are referred to the stator. The exciting circuit is accounted for as in a single cage motor. If the magnetizing current ( $I_0$ ) is neglected, then the circuit is simplified to that shown in Fig. (8.38 (ii)).



From the equivalent circuit, the performance of the motor can be predicted. Total impedance as referred to stator is

$$Z_{o1} = R_1 + j X_1 + \frac{1}{1/Z'_i + 1/Z'_o} = R_1 + j X_1 + \frac{Z'_i Z'_o}{Z'_i + Z'_o}$$

Chapter (9)

# **Single-Phase Motors**

# Introduction

As the name suggests, these motors are used on single-phase supply. Singlephase motors are the most familiar of all electric motors because they are extensively used in home appliances, shops, offices etc. It is true that singlephase motors are less efficient substitute for 3-phase motors but 3-phase power is normally not available except in large commercial and industrial establishments. Since electric power was originally generated and distributed for lighting only, millions of homes were given single-phase supply. This led to the development of single-phase motors. Even where 3-phase mains are present, the single-phase supply may be obtained by using one of the three lines and the neutral. In this chapter, we shall focus our attention on the construction, working and characteristics of commonly used single-phase motors.

# 9.1 Types of Single-Phase Motors

Single-phase motors are generally built in the fractional-horsepower range and may be classified into the following four basic types:

- 1. Single-phase induction motors
  - (i) split-phase type

(ii) capacitor type

- (iii) shaded-pole type
- 2. A.C. series motor or universal motor
- 3. Repulsion motors
  - (i) Repulsion-start induction-run motor
  - (ii) Repulsion-induction motor
- 4. Synchronous motors
  - (i) Reluctance motor (ii) Hysteresis motor

# 9.2 Single-Phase Induction Motors

A single phase induction motor is very similar to a 3-phase squirrel cage induction motor. It has (i) a squirrel-cage rotor identical to a 3-phase motor and (ii) a single-phase winding on the stator.

Unlike a 3-phase induction motor, a single-phase induction motor is not selfstarting but requires some starting means. The single-phase stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel-cage rotor. However, if the rotor of a single-phase motor is rotated in one direction by some mechanical means, it will continue to run in the direction of rotation. As a matter of fact, the rotor quickly accelerates until it reaches a speed slightly below the synchronous speed. Once the motor is running at this speed, it will continue to rotate even though single-phase current is flowing through the stator winding. This method of starting is generally not convenient for large motors. Nor can it be employed fur a motor located at some inaccessible spot.

Fig. (9.1) shows single-phase induction motor having a squirrel cage rotor and a singlephase distributed stator winding. Such a motor inherently docs not develop any starting torque and, therefore, will not start to rotate if the stator winding is connected to single-phase a.c. supply. However, if the rotor is started by auxiliary means, the motor



will quickly attain me final speed. This strange behaviour of single-phase induction motor can be explained on the basis of double-field revolving theory.

# 9.3 Double-Field Revolving Theory

The double-field revolving theory is proposed to explain this dilemma of no torque at start and yet torque once rotated. This theory is based on the fact that an alternating sinusoidal flux ( $\phi = \phi_m \cos \omega t$ ) can be represented by two revolving fluxes, each equal to one-half of the maximum value of alternating flux (i.e.,  $\phi_m/2$ ) and each rotating at synchronous speed (N<sub>s</sub> = 120 f/P,  $\omega = 2\pi f$ ) in opposite directions.

The above statement will now be proved. The instantaneous value of flux due to the stator current of a single-phase induction motor is given by;

$$\phi = \phi_m \cos \omega t$$

Consider two rotating magnetic fluxes  $\phi_1$  and  $\phi_2$  each of magnitude  $\phi_m/2$  and rotating in opposite directions with angular velocity  $\omega$  [See Fig. (9.2)]. Let the two fluxes start rotating from OX axis at



t = 0. After time t seconds, the angle through which the flux vectors have rotated is at. Resolving the flux vectors along-X-axis and Y-axis, we have,

Total X-component 
$$= \frac{\phi_m}{2} \cos \omega t + \frac{\phi_m}{2} \cos \omega t = \phi_m \cos \omega t$$
  
Total Y-component  $= \frac{\phi_m}{2} \sin \omega t - \frac{\phi_m}{2} \sin \omega t = 0$   
Resultant flux,  $\phi = \sqrt{(\phi_m \cos \omega t)^2 + 0^2} = \phi_m \cos \omega t$ 

Thus the resultant flux vector is  $\phi = \phi_m \cos \omega t$  along X-axis. Therefore, an alternating field can be replaced by two relating fields of half its amplitude rotating in opposite directions at synchronous speed. Note that the resultant vector of two revolving flux vectors is a stationary vector that oscillates in length with time along X-axis. When the rotating flux vectors are in phase [See Fig.



(9.3 (i))], the resultant vector is  $\phi = \phi_m$ ; when out of phase by 180° [See Fig. (9.3 (ii))], the resultant vector  $\phi = 0$ .

Let us explain the operation of single-phase induction motor by double-field revolving theory.

#### (i) Rotor at standstill

Consider the case that the rotor is stationary and the stator winding is connected to a single-phase supply. The alternating flux produced by the stator winding can be presented as the sum of two rotating fluxes  $\phi_1$  and  $\phi_2$ , each equal to one half of the maximum value of alternating flux and each rotating at synchronous speed (N<sub>s</sub> = 120 f/P) in opposite directions as shown in Fig. (9.4 (i)). Let the flux  $\phi_1$  rotate in anti clockwise direction and flux  $\phi_2$  in clockwise direction. The flux  $\phi_1$  will result in the production of torque T<sub>1</sub> in the anti clockwise direction and flux  $\phi_2$  will result in the production of torque T<sub>2</sub> In the clockwise direction. At standstill, these two torques are equal and opposite and the net torque developed is zero. Therefore, single-phase induction motor is not self-starting. This fact is illustrated in Fig. (9.4 (ii)).

Note that each rotating field tends to drive the rotor in the direction in which the field rotates. Thus the point of zero slip for one field corresponds to 200% slip for the other as explained later. The value of 100% slip (standstill condition) is the same for both the fields.



#### (ii) Rotor running

...

Now assume that the rotor is started by spinning the rotor or by using auxiliary circuit, in say clockwise direction. The flux rotating in the clockwise direction is the forward rotating flux ( $\phi_f$ ) and that in the other direction is the backward rotating flux ( $\phi_b$ ). The slip w.r.t. the forward flux will be

$$s_{f} = \frac{N_{s} - N}{N_{s}} = s$$

where  $N_s =$  synchronous speed

N = speed of rotor in the direction of forward flux

The rotor rotates opposite to the rotation of the backward flux. Therefore, the slip w.r.t. the backward flux will be

$$s_{b} = \frac{N_{s} - (-N)}{N_{s}} = \frac{N_{s} + N}{N_{s}} = \frac{2N_{s} - N_{s} + N}{N_{s}}$$
$$= \frac{2N_{s}}{N_{s}} - \frac{(N_{s} - N)}{N_{s}} = 2 - s$$
$$s_{b} = 2 - s$$

Thus fur forward rotating flux, slip is s (less than unity) and for backward rotating flux, the slip is 2 - s (greater than unity). Since for usual rotor resistance/reactance ratios, the torques at slips of less than unity arc greater than those at slips of more than unity, the resultant torque will be in the direction of the rotation of the forward flux. Thus if the motor is once started, it will develop net torque in the direction in which it has been started and will function as a motor.

Fig. (9.5) shows the rotor circuits for the forward and backward rotating fluxes. Note that  $r_2 = R_2/2$ , where  $R_2$  is the standstill rotor resistance i.e.,  $r_2$  is equal to half the standstill rotor resistance. Similarly,  $x_2 = X_2/2$  where  $X_2$  is the standstill rotor reactance. At standstill, s = 1 so that impedances of the two circuits are equal. Therefore, rotor currents are equal i.e.,  $I_{2f} = I_{2b}$ . However, when the rotor rotates, the impedances of the two rotor circuits are unequal and the rotor current  $I_{2b}$  is higher (and also at a lower power factor) than the rotor current  $I_{2f}$ . Their m.m.f.s, which oppose the stator m.m.f.s, will result in a reduction of the backward rotating flux. Consequently, as speed increases, the forward flux increases, reducing the opposing torque. The motor-quickly accelerates to the final speed.



**Fig.(9.5)** 

#### 9.4 Making Single-Phase Induction Motor Self-Starting

The single-phase induction motor is not selfstarting and it is undesirable to resort to mechanical spinning of the shaft or pulling a belt to start it. To make a single-phase induction motor self-starting. we should somehow produce a revolving stator magnetic field. This may be achieved by converting a single-phase supply into two-phase supply through the use of an additional winding. When the motor attains sufficient speed, the starting means (i.e., additional winding) may be removed depending upon the type of the motor. As a matter of fact, single-phase induction motors are classified and named



according to the method employed to make them self-starting.

(i) **Split-phase motors**-started by two phase motor action through the use of an auxiliary or starting winding.

- (ii) **Capacitor motors**-started by two-phase motor action through the use of an auxiliary winding and a capacitor.
- (iii) **Shaded-pole motors**-started by the motion of the magnetic field produced by means of a shading coil around a portion of the pole structure.

## 9.5 Rotating Magnetic Field From 2-Phase Supply

As with a 3-phase supply, a 2-phase balanced supply also produces a rotating magnetic field of constant magnitude. With the exception of the shaded-pole motor, all single-phase induction motors are started as 2-phase machine. Once so started, the motor will continue to run on single-phase supply.

Let us see how 2-phase supply produces a rotating magnetic field of constant magnitude. Fig. (9.10 (i)) shows 2-pole, 2-phase winding. The phases X and Y are energized from a two-phase source and currents in these phases arc indicated as  $I_x$  and  $I_y$  [See Fig. (9.10 (ii))]. Referring to Fig. (9.10 (ii)), the fluxes produced by these currents arc given by;

$$\phi_{\rm Y} = \phi_{\rm m} \sin \omega t$$
 and  $\phi_{\rm X} = \phi_{\rm m} \sin(\omega t + 90^\circ) = \phi_{\rm m} \cos \omega t$ 

Here  $\phi_m$  is the maximum flux due to either phase. We shall now prove that this 2-phase supply produces a rotating magnetic field of constant magnitude equal to  $\phi_m$ .

(i) At instant 1 [See (Fig. 9.10 (ii)) and Fig. (9.10 (iii))], the current is zero in phase Y and maximum in phase X. With the current in the direction shown, a resultant flux is established toward the right. The magnitude of the resultant flux is constant and is equal to  $\phi_m$  as proved under:

At instant 1, 
$$\omega t = 0^{\circ}$$
  $\therefore$   $\phi_{Y} = 0$  and  $\phi_{X} = \phi_{m}$  Fig.(9.7)  
 $\therefore$  Resultant flux,  $\phi_{r} = \sqrt{\phi_{X}^{2} + \phi_{Y}^{2}} = \sqrt{(\phi_{m})^{2} + (0)^{2}} = \phi_{m}$ 

(ii) At instant 2 [See Fig. (9.10 (ii)) and Fig. (9.10 (iii))], the current is still in the same direction in phase X and an equal current flowing in phase Y. This establishes a resultant flux of the same value (i.e.,  $\phi_r = \phi_m$ ) as proved under:

At instant 2, 
$$\omega t = 45^{\circ}$$
  $\therefore \quad \phi_{Y} = \frac{\phi_{m}}{\sqrt{2}} \text{ and } \phi_{X} = \frac{\phi_{m}}{\sqrt{2}}$   
 $\therefore \quad \text{Resultant flux, } \phi_{r} = \sqrt{(\phi_{X})^{2} + (\phi_{Y})^{2}}$   
 $= \sqrt{\left(\frac{\phi_{m}}{\sqrt{2}}\right)^{2} + \left(\frac{\phi_{m}}{\sqrt{2}}\right)^{2}} = \phi_{m}$ 
Fig.(9.8)

Note that resultant flux has the same value (i.e.  $\phi_m$ ) but turned 45° clockwise from position 1.

Fig.(9.9)

(iii) At instant 3 [See Fig. (9.10 (ii)) and Fig. (9.10 (iii))], the current in phase. X has decreased to zero and current in phase Y has increased to maximum. This establishes a resultant flux downward as proved under:





At instant 3,  $\omega t = 90^{\circ}$   $\therefore$   $\phi_{Y} = \phi_{m}$  and  $\phi_{X} = 0$ 

: 
$$\phi_{\rm r} = \sqrt{\phi_{\rm X}^2 + (\phi_{\rm Y})^2} = \sqrt{(0)^2 + (\phi_{\rm m})^2} = \phi_{\rm m}$$

Note that resultant flux has now turned 90° clockwise from position 1. The reader may note that in the three instants considered above, the resultant flux is constant and is equal to  $\phi_m$ . However, this constant resultant flux is shining its position (clockwise in this case). In other words, the rotating flux is produced. We shall continue to consider other instants to prove this fact.

(iv) At instant 4 [See Fig. (9.10 (ii)) and Fig. (9.10 (iii))], the current in phase X has reversed and has the same value as that of phase Y. This establishes a resultant flux equal to  $\phi_m$  turned 45° clockwise from position 3.

At instant 4, 
$$\omega t = 135^{\circ}$$
  $\therefore$   $\phi_{Y} = \frac{\phi_{m}}{\sqrt{2}}$  and  $\phi_{X} = \frac{\phi_{m}}{\sqrt{2}}$   
 $\therefore$   $\phi_{r} = \sqrt{\phi_{X}^{2} + \phi_{Y}^{2}} = \sqrt{\left(-\frac{\phi_{m}}{\sqrt{2}}\right)^{2} + \left(\frac{\phi_{m}}{\sqrt{2}}\right)^{2}} = \phi_{m}$  Fig.(9.11)

(v) At instant 5 [See Fig. (9.10 (ii)) and Fig. (9.10 (iii))], the current in phase X is maximum and in phase Y is zero. This establishes a resultant flux equal to  $\phi_m$  toward left (or 90° clockwise from position 3).

At instant 5, 
$$\omega t = 180^{\circ}$$
  $\therefore$   $\phi_{Y} = 0$  and  $\phi_{X} = -\phi_{m}$   
 $\therefore$   $\phi_{r} = \sqrt{\phi_{X}^{2} + \phi_{Y}^{2}} = \sqrt{(-\phi_{m})^{2} + (0)^{2}} = \phi_{m}$   
Fig.(9.12)

(vi) Diagrams 6, 7, and 8 [See Fig. (9.10 (iii))] indicate the direction of the resultant flux during the remaining successive instants.

It follows from the above discussion that a 2-phase supply produces a rotating magnetic field of constant value (=  $\phi_m$  the maximum value of one of the fields).

*Note*: If the two windings arc displaced  $90^{\circ}$  electrical but produce fields that are not equal and that are not  $90^{\circ}$  apart in time, the resultant field is still rotating but is not constant in magnitude. One effect of this nonuniform rotating field is the production of a torque that is non-uniform and that, therefore, causes noisy operation of the motor. Since 2-phase operation ceases once the motor is started, the operation of the motor then becomes smooth.

## 9.6 Split-Phase Induction Motor

The stator of a split-phase induction motor is provided with an auxiliary or starting winding S in addition to the main or running winding M. The starting winding is located 90° electrical from the main winding [See Fig. (9.13 (i))] and operates only during the brief period when the motor starts up. The two windings are so resigned that the starting winding S has a high resistance and relatively small reactance while the main winding M has relatively low resistance and large reactance as shown in the schematic connections in Fig. (9.13 (ii)). Consequently, the currents flowing in the two windings have reasonable phase difference c  $(25^{\circ} \text{ to } 30^{\circ})$  as shown in the phasor diagram in Fig. (9.13 (iii)).



**Fig.(9.13)** 

#### Operation

- (i) When the two stator windings are energized from a single-phase supply, the main winding carries current  $I_m$  while the starting winding carries current  $I_s$ .
- (ii) Since main winding is made highly inductive while the starting winding highly resistive, the currents  $I_m$  and  $I_s$  have a reasonable phase angle a (25° to 30°) between them as shown in Fig. (9.13 (iii)). Consequently, a weak revolving field approximating to that of a 2-phase machine is produced which starts the motor. The starting torque is given by;

$$T_s = kI_m I_s \sin \alpha$$

where k is a constant whose magnitude depends upon the design of the motor.

(iii) When the motor reaches about 75% of synchronous speed, the centrifugal switch opens the circuit of the starting winding. The motor then operates as a single-phase induction motor and continues to accelerate till it reaches the

normal speed. The normal speed of the motor is below the synchronous speed and depends upon the load on the motor.

#### Characteristics

- (i) The sinning torque is 15 to 2 times the full-loud torque mid (lie starting current is 6 to 8 times the full-load current.
- (ii) Due to their low cost, split-phase induction motors are most popular singlephase motors in the market.
- (iii) Since the starting winding is made of fine wire, the current density is high and the winding heats up quickly. If the starting period exceeds 5 seconds, the winding may burn out unless the motor is protected by built-in-thermal relay. This motor is, therefore, suitable where starting periods are not frequent.
- (iv) An important characteristic of these motors is that they are essentially constant-speed motors. The speed variation is 2-5% from no-load to full-load.
- (v) These motors are suitable where a moderate starting torque is required and where starting periods are infrequent e.g., to drive:

(a) fans (b) washing machines (c) oil burners (d) small machine tools etc.

The power rating of such motors generally lies between 60 W and 250 W.

# 9.7 Capacitor-Start Motor

The capacitor-start motor is identical to a split-phase motor except that the starting winding has as many turns as the main winding. Moreover, a capacitor C is connected in series with the starting winding as shown in Fig. (9.14 (i)). The value of capacitor is so chosen that  $I_s$  leads  $I_m$  by about 80° (i.e.,  $\alpha \simeq 80^\circ$ ) which is considerably greater than 25° found in split-phase motor [See Fig. (9.14 (ii))]. Consequently, starting torque ( $T_s = k I_m I_s \sin \alpha$ ) is much more than that of a split-phase motor Again, the starting winding is opened by the centrifugal switch when the motor attains about 75% of synchronous speed. The motor then operates as a single-phase induction motor and continues to accelerate till it reaches the normal speed.

## Characteristics

- (i) Although starting characteristics of a capacitor-start motor are better than those of a split-phase motor, both machines possess the same running characteristics because the main windings are identical.
- (ii) The phase angle between the two currents is about 80° compared to about 25° in a split-phase motor. Consequently, for the same starting torque, the current in the starting winding is only about half that in a split-phase motor. Therefore, the starting winding of a capacitor start motor heats up less

quickly and is well suited to applications involving either frequent or prolonged starting periods.



**Fig.(9.14)** 

(iii) Capacitor-start motors are used where high starting torque is required and where the starting period may be long e.g., to drive:

(a) compressors (b) large fans (c) pumps (d) high inertia loads

The power rating of such motors lies between 120 W and 7-5 kW.

#### 9.8 Capacitor-Start Capacitor-Run Motor

This motor is identical to a capacitor-start motor except that starting winding is not opened after starting so that both the windings remain connected to the supply when running as well as at starting. Two designs are generally used.

(i) In one design, a single capacitor C is used for both starting and running as shown in Fig.(9.15 (i)). This design eliminates the need of a centrifugal switch and at the same time improves the power factor and efficiency of the motor.



(ii) In the other design, two capacitors  $C_1$  and  $C_2$  are used in the starting winding as shown in Fig. (9.15 (ii)). The smaller capacitor  $C_1$  required for optimum running conditions is permanently connected in series with the

starting winding. The much larger capacitor  $C_2$  is connected in parallel with  $C_1$  for optimum starting and remains in the circuit during starting. The starting capacitor  $C_1$  is disconnected when the motor approaches about 75% of synchronous speed. The motor then runs as a single-phase induction motor.

#### Characteristics

- (i) The starting winding and the capacitor can be designed for perfect 2-phase operation at any load. The motor then produces a constant torque and not a pulsating torque as in other single-phase motors.
- (ii) Because of constant torque, the motor is vibration free and can be used in:(a) hospitals (6) studios and (c) other places where silence is important.

# 9.9 Shaded-Pole Motor

The shaded-pole motor is very popular for ratings below 0.05 H.P. ( $\simeq 40$  W) because of its extremely simple construction. It has salient poles on the stator excited by single-phase supply and a squirrelcage rotor as shown in Fig. (9.16). A portion of each pole is surrounded by a short-circuited turn of copper strip called shading coil.



## Operation

The operation of the motor can be understood by referring to Fig. (9.17) which shows one pole of the motor with a shading coil.

- (i) During the portion OA of the alternating-current cycle [See Fig. (9.17)], the flux begins to increase and an e.m.f. is induced in the shading coil. The resulting current in the shading coil will be in such a direction (Lenz's law) so as to oppose the change in flux. Thus the flux in the shaded portion of the pole is weakened while that in the unshaded portion is strengthened as shown in Fig. (9.17 (ii)).
- (ii) During the portion AB of the alternating-current cycle, the flux has reached almost maximum value and is not changing. Consequently, the flux distribution across the pole is uniform [See Fig. (9.17 (iii))] since no current is flowing in the shading coil. As the flux decreases (portion BC of the alternating current cycle), current is induced in the shading coil so as to oppose the decrease in current. Thus the flux in the shaded portion of the

pole is strengthened while that in the unshaded portion is weakened as shown in Fig. (9.17 (iv)).



- (iii) The effect of the shading coil is to cause the field flux to shift across the pole face from the unshaded to the shaded portion. This shifting flux is like a rotating weak field moving in the direction from unshaded portion to the shaded portion of the pole.
- (iv) The rotor is of the squirrel-cage type and is under the influence of this moving field. Consequently, a small starting torque is developed. As soon as this torque starts to revolve the rotor, additional torque is produced by single-phase induction-motor action. The motor accelerates to a speed slightly below the synchronous speed and runs as a single-phase induction motor.

#### Characteristics

- (i) The salient features of this motor are extremely simple construction and absence of centrifugal switch.
- (ii) Since starting torque, efficiency and power factor are very low, these motors are only suitable for low power applications e.g., to drive:(a) small fans (6) toys (c) hair drivers (d) desk fans etc.

The power rating of such motors is upto about 30 W.

# 9.10 Equivalent Circuit of Single-Phase Induction Motor

It was stated earlier that when the stator of a single-phase induction motor is con heeled to single-phase supply, the stator current produces a pulsating flux that is equivalent to two-constant-amplitude fluxes revolving in opposite directions at the synchronous speed (double-field revolving theory). Each of these fluxes induces currents in the rotor circuit and produces induction motor action similar to that in a 3-phase induction motor Therefore, a single-phase induction motor can to imagined to be consisting of two motors, having a common stator winding but with their respective rotors revolving in opposite directions. Each rotor has resistance and reactance half the actual rotor values.

- Let  $R_1$  = resistance of stator winding
  - $X_1$  = leakage reactance of stator winding
  - $X_m$  = total magnetizing reactance
  - $R'_2$  = resistance of the rotor referred to the stator
  - $X'_2$  = leakage reactance of the rotor referred to the stator

revolving theory.

(i) At standstill. At standstill, the motor is simply a transformer with its secondary short-circuited. Therefore, the equivalent circuit of single-phase motor at standstill will be as shown in Fig. (9.18). The double-field revolving theory suggests that characteristics associated with each revolving field will be just one-half of the characteristics associated with the actual total flux. Therefore, each rotor has resistance and reactance equal to  $R'_2/2$  and  $X'_2/2$  respectively. Each rotor is associated with half the



Fig.(9.18)

total magnetizing reactance. Note that in the equivalent circuit, the core loss has been neglected. However, core loss can be represented by an equivalent resistance in parallel with the magnetizing reactance.

$$\begin{array}{lll} Now & E_f = 4.44 \ f \ N \ \phi_f; & E_b = 4.44 \ f \ N \ \phi_b \end{array}$$
  
At standstill,  $\phi_f = \phi_b$ . Therefore,  $E_f = E_b$ .
  
 $V_1 \simeq E_f + E_b = I_1 Z_f + I_1 Z_b$ 
  
where  $Z_f = \mbox{impedance of forward parallel branch}$ 

 $Z_b =$  impedance of backward parallel branch

(ii) **Rotor running**. Now consider that the motor is pinning at some speed in the direction of the forward revolving field, the slip being s. The rotor current produced by the forward field will have a frequency sf where f is the stator frequency. Also, the rotor current produced by the backward field will have a frequency of (2 - s)f. Fig. (9.19) shows the equivalent circuit of a single-phase induction motor when the rotor is rotating at slip s. It is clear, from the equivalent circuit that under running conditions,  $E_f$  becomes much greater than  $E_b$  because the term  $R'_2/2s$  increases very much as s tends towards zero. Conversely,  $E^{\Lambda}$  falls because the term  $R'_2/2(2 - s)$  decreases since (2 - s) tends toward 2. Consequently, the forward field increases, increasing the driving torque while the backward field decreases reducing the opposing torque.



Fig.(9.19)

Total impedance of the circuit .is given by;

$$Z_{r} = Z_{1} + Z_{f} + Z_{b}$$

$$Z_{1} = R_{1} + j X_{1}$$

$$X \quad (R')$$

7 . 7 . 7

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$$Z_{f} = \frac{j\frac{X_{m}}{2}\left(\frac{R'_{2}}{2s} + j\frac{X'_{2}}{2}\right)}{\frac{R'_{2}}{2s} + j\left(\frac{X_{m}}{2} + \frac{X'_{2}}{2}\right)}$$
$$Z_{b} = \frac{j\frac{X_{m}}{2}\left(\frac{R'_{2}}{2(2-s)} + j\frac{X'_{2}}{2}\right)}{\frac{R'_{2}}{2(2-s)} + j\left(\frac{X_{m}}{2} + \frac{X'_{2}}{2}\right)}$$
$$\therefore \quad I_{1} = V_{1}/Z_{r}$$

9.11 A.C. Series Motor or Universal Motor

A d.c. series motor will rotate in the same direction regardless of the polarity of the supply. One can expect that a d.c. series motor would also operate on a single-phase supply. It is then called an a.c. series motor. However, some changes must be made in a d.c. motor that is to operate satisfactorily on a.c. supply. The changes effected are:

- (i) The entire magnetic circuit is laminated in order to reduce the eddy current loss. Hence an a.c. series motor requires a more expensive construction than a d.c. series motor.
- (ii) The series field winding uses as few turns as possible to reduce the reactance of the field winding to a minimum. This reduces the voltage drop across the field winding.
- (iii) A high field flux is obtained by using a low-reluctance magnetic circuit.
- (iv) There is considerable sparking between the brushes and the commutator when the motor is used on a.c. supply. It is because the alternating flux establishes high currents in the coils short-circuited by the brushes. When the short-circuited coils break contact from the commutator, excessive sparking is produced. This can be eliminated by using high-resistance leads to connect the coils to the commutator segments.

## Construction

The construction of en a.c. series motor is very similar to a d.c. series motor

except that above modifications are incorporated [See Fig. (9.20)]. Such a motor can be operated either on a.c. or d.c. supply and the resulting torque-speed curve is about the same in each case. For this reason, it is sometimes called a universal motor.

## Operation

When the motor is connected to an a.c. supply, the same alternating current flows through the field and armature windings. The field winding produces an alternating flux  $\phi$  that reacts with the current flowing in



the armature to produce a torque. Since both armature current and flux reverse simultaneously, the torque always acts in the same direction. It may be noted that no rotating flux is produced in this type of machines; the principle of operation is the same as that of a d.c. series motor.

## Characteristics

The operating characteristics of an a.c. series motor are similar to those of a d.c. series motor.

- (i) The speed increases to a high value with a decrease in load. In very small series motors, the losses are usually large enough at no load that limit the speed to a definite value (1500 15,000 r.p.m.).
- (ii) The motor torque is high for large armature currents, thus giving a high starting torque.
- (iii) At full-load, the power factor is about 90%. However, at starting or when carrying an overload, the power factor is lower.

## Applications

The fractional horsepower a.c. series motors have high-speed (and corresponding small size) and large starting torque. They can, therefore, be used to drive:

- (a) high-speed vacuum cleaners
- (b) sewing machines

(c) electric shavers

(d) drills

(e) machine tools etc.

# 9.12 Single-Phase Repulsion Motor

A repulsion motor is similar to an a.c. series motor except that:

- (i) brushes are not connected to supply but are short-circuited [See Fig. (9.21)]. Consequently, currents are induced in the armature conductors by transformer action.
- (ii) the field structure has non-salient pole construction.

By adjusting the position of short-circuited brushes on the commutator, the starting torque can be developed in the motor.

#### Construction

The field of stator winding is wound like the main winding of a split-phase motor and is connected directly to a single-phase source. The armature or rotor is similar to a d.c. motor armature with drum type winding connected to a commutator (not shown in the figure). However, the brushes are not connected to supply but are connected to each other or short-circuited. Short-circuiting the brushes effectively makes the rotor into a type of squirrel cage. The major difficulty with an ordinary single-phase induction motor is the low starting torque. By using a commutator motor with brushes short-circuited, it is possible to vary the starting torque by changing the brush axis. It has also better power factor than the conventional single-phase motor.



#### **Principle of operation**

The principle of operation is illustrated in Fig. (9.21) which shows a two-pole repulsion motor with its two short-circuited brushes. The two drawings of Fig. (9.21) represent a time at which the field current is increasing in the direction shown so that the left-hand pole is N-pole and the right-hand pole is S-pole at the instant shown.

(i) In Fig. (9.21 (i)), the brush axis is parallel to the stator field. When the stator winding is energized from single-phase supply, e.m.f. is induced in the armature conductors (rotor) by induction. By Lenz's law, the direction of the e.m.f. is such that the magnetic effect of the resulting armature currents will oppose the increase in flux. The direction of current in armature conductors will be as shown in Fig. (9.21 (i)). With the brush axis in the position shown in Fig. (9.21 (i)), current will flow from brush B to

brush A where it enters the armature and flows back to brush B through the two paths ACB and ADB. With brushes set in this position, half of the armature conductors under the N-pole carry current inward and half carry current outward. The same is true under S-pole. Therefore, as much torque is developed in one direction as in the other and the armature remains stationary. The armature will also remain stationary if the brush axis is perpendicular to the stator field axis. It is because even then net torque is zero.

- (ii) If the brush axis is at some angle other than 0° or 90° to the axis of the stator field, a net torque is developed on the rotor and the rotor accelerates to its final speed. Fig. (9.21 (ii)) represents the motor at the same instant as that in Fig. (9.21 (i)) but the brushes have been shifted clockwise through some angle from the stator field axis. Now e.m.f. is still induced in the direction indicated in Fig. (9.21 (i)) and current flows through the two paths of the armature winding from brush A to brush B. However, because of the new brush positions, the greater part of the conductors under the N-pole carry current in one direction while the greater part of conductors under S-pole carry current in the opposite direction. With brushes in the position shown in Fig. (9.21 (ii), torque is developed in the clockwise direction and the rotor quickly attains the final speed.
- (iii) The direction of rotation of the rotor depends upon the direction in which the brushes are shifted. If the brushes are shifted in clockwise direction from the stator field axis, the net torque acts in the clockwise direction and the rotor accelerates in the clockwise direction. If the brushes are shifted in anti-clockwise



direction as in Fig. (9.22). the armature current under the pole faces is reversed and the net torque is developed in the anti-clockwise direction. Thus a repulsion motor may be made to rotate in either direction depending upon the direction in which the brushes are shifted.

(iv) The total armature torque in a repulsion motor can be shown to be

 $T_a \propto \sin 2\alpha$ 

where  $\alpha$  = angle between brush axis and stator field axis For maximum torque,  $2\alpha = 90^{\circ}$  or  $\alpha = 45^{\circ}$  Thus adjusting  $\alpha$  to 45° at starting, maximum torque can be obtained during the starting period. However,  $\alpha$  has to be adjusted to give a suitable running speed.

#### Characteristics

- (i) The repulsion motor has characteristics very similar to those of an a.c. series motor i.e., it has a high starting torque and a high speed at no load.
- (ii) The speed which the repulsion motor develops for any given load will depend upon the position of the brushes.
- (iii) In comparison with other single-phase motors, the repulsion motor has a high starring torque and relatively low starting current.

# 9.13 Repulsion-Start Induction-Run Motor

Sometimes the action of a repulsion motor is combined with that of a singlephase induction motor to produce repulsion-start induction-run motor (also called repulsion-start motor). The machine is started as a repulsion motor with a corresponding high starting torque. At some predetermined speed, a centrifugal device short-circuits the commutator so that the machine then operates as a single-phase induction motor.

The repulsion-start induction-run motor has the same general construction of a repulsion motor. The only difference is that in addition to the basic repulsion-motor construction, it is equipped with a centrifugal device fitted on the armature shaft. When the motor reaches 75% of its full pinning speed, the centrifugal device forces a short-circuiting ring to come in contact with the inner surface of the commutator. This snort-circuits all the commutator bars. The rotor then resembles squirrel-cage type and the motor runs as a single-phase induction motor. At the same time, the centrifugal device raises the brushes from the commutator which reduces the wear of the brushes and commutator as well as makes the operation quiet.

## Characteristics

- (i) The starting torque is 2.5 to 4.5 times the full-load torque and the starting current is 3.75 times the full-load value.
- (ii) Due to their high starting torque, repulsion-motors were used to operate devices such as refrigerators, pumps, compressors etc.

However, they posed a serious problem of maintenance of brushes, commutator arid the centrifugal device. Consequently, manufacturers have stopped making them in view of the development of capacitor motors which are small in size, reliable and low-priced.

# 9.14 Repulsion-Induction Motor

The repulsion-induction motor produces a high starting torque entirely due to repulsion motor action. When running, it functions through a combination of induction-motor and repulsion motor action.

## Construction

Fig. (9.23) shows the connections of a 4-pole repulsion-induction motor for 230 V operation. It consists of a stator and a rotor (or armature).

- (i) The stator carries a single distributed winding fed from single-phase supply.
- (ii) The rotor is provided with two independent windings placed one inside the other. The inner winding is a squirrel-cage winding with rotor bars permanently short-circuited. Placed over the squirrel cage winding is a repulsion commutator armature winding. The repulsion winding is connected to a commutator on which ride short-circuited brushes. There is no centrifugal device and the repulsion winding functions at all times.



**Fig.(9.23)** 

# Operation

- (i) When single-phase supply is given to the stator winding, the repulsion winding (i.e., outer winding) is active. Consequently, the motor starts as a repulsion motor with a corresponding high starting torque.
- (ii) As the motor speed increases, the current shifts from the outer to inner winding due to the decreasing impedance of the inner winding with increasing speed. Consequently, at running speed, the squirrel cage winding carries the greater part of rotor current. This shifting of repulsionmotor action to induction-motor action is thus achieved without any switching arrangement.
- (iii) It may be seen that the motor starts as a repulsion motor. When running, it functions through a combination of principle of induction and repulsion; the former being predominant.

## Characteristics

(i) The no-load speed of a repulsion-induction motor is somewhat above the synchronous speed because of the effect of repulsion winding. However,

the speed at full-load is slightly less than the synchronous speed as in an induction motor.

- (ii) The speed regulation of the motor is about 6%.
- (iii) The starting torque is 2.25 to 3 times the full-load torque; the lower value being for large motors. The starting current is 3 to 4 times the full-load current.

This type of motor is used for applications requiring a high starting torque with essentially a constant running speed. The common sizes are 0.25 to 5 H.P.

# 9.15 Single-Phase Synchronous Motors

Very small single-phase motors have been developed which run at true synchronous speed. They do not require d.c. excitation for the rotor. Because of these characteristics, they are called unexcited single-phase synchronous motors. The most commonly used types are:

(i) Reluctance motors (ii) Hysteresis motors

The efficiency and torque-developing ability of these motors is low; The output of most of the commercial motors is only a few watts.

# 9.16 Reluctance Motor

It is a single-phase synchronous motor which does not require d.c. excitation to the rotor. Its operation is based upon the following principle:

Whenever a piece of ferromagnetic material is located in a magnetic field; a force is exerted on the material, tending to align the material so that reluctance of the magnetic path that passes through the material is minimum.



**Fig.(9.24)** 

## Construction

A reluctance motor (also called synchronous reluctance motor) consists of:

- (i) **a stator** carrying a single-phase winding along with an auxiliary winding to produce a synchronous-revolving magnetic field.
- (ii) a squirrel-cage rotor having unsymmetrical magnetic construction. This is achieved by symmetrically removing some of the teeth from the squirrel-cage rotor to produce salient poles on the rotor. As shown in Fig. (9.24 (i)), 4 sailent poles have been produced on me rotor. The salient poles created on the rotor must be equal to the poles on the stator.

Note that rotor salient poles offer low reductance to the stator flux and, therefore, become strongly magnetized.

## Operation

- (i) When single-phase stator having an auxiliary winding is energized, a synchronously-revolving field is produced. The motor starts as a standard squirrel-cage induction motor and will accelerate to near its synchronous speed.
- (ii) As the rotor approaches synchronous speed, the rotating stator flux will exert reluctance torque on the rotor poles tending to align the salient-pole axis with the axis of the rotating field. The rotor assumes a position where its salient poles lock with the poles of the revolving field [See Fig. (9.24 (ii))]. Consequently, the motor will continue to run at the speed of revolving flux i.e., at the synchronous speed.
- (iii) When we apply a mechanical load, the rotor poles fall slightly behind the stator poles, while continuing to turn at synchronous speed. As the load on the motor is increased, the mechanical angle between the poles increases progressively. Nevertheless, magnetic attraction keeps the rotor locked to the rotating flux. If the load is increased beyond the amount under which the reluctance torque can maintain synchronous speed, the rotor drops out of step with the revolving field. The speed, then, drops to some value at which the slip is sufficient to develop the necessary torque to drive the load by induction-motor action.

#### Characteristics

- (i) These motors have poor torque, power factor and efficiency.
- (ii) These motors cannot accelerate high-inertia loads to synchronous speed.
- (iii) The pull-in and pull-out torques of such motors are weak.

Despite the above drawbacks, the reluctance motor is cheaper than any other type of synchronous motor. They are widely used for constant-speed applications such as timing devices, signalling devices etc.

# 9.17 Hysteresis Motor

It is a single-phase motor whose operation depends upon the hysteresis effect i.e., magnetization produced in a ferromagnetic material lags behind the magnetizing force.

#### **Construction** It consists of:

- (i) **a stator** designed to produce a synchronously-revolving field from a single-phase supply. This is accomplished by using permanent-split capacitor type construction. Consequently, both the windings (i.e., starting as well as main winding) remain connected in the circuit during running operation as well as at starting. The value of capacitance is so adjusted as to result in a flux revolving at synchronous speed.
- (ii) **a rotor** consisting of a smooth cylinder of magnetically hard steel, without winding or teeth.

## Operation

- (i) When the stator is energized from a single-phase supply, a synchronouslyrevolving field (assumed in anti-clockwise direction) is produced due to split-phase operation.
- (ii) The revolving stator flux magnetizes the rotor. Due to hysteresis effect, the axis of magnetization of rotor will lag behind the axis of stator field by hysteresis lag angle a as shown in Fig. (9.25). Thus the rotor and stator poles are locked. If the rotor is stationary, the starting torque produced is given by:

$$T_s \propto \phi_s \phi_r \sin \alpha$$

where  $\phi_s = \text{stator flux}$ .

 $\phi_r$  = rotor flux.

From now onwards, the rotor accelerates to synchronous speed with a uniform torque.

(iii) After reaching synchronism, the motor continues to run at synchronous speed and adjusts its torque angle so as to develop the torque required by the load.

## Characteristics

- (i) A hysteresis motor can synchronize any load which it can accelerate, no matter how great the inertia. It is because the torque is uniform from standstill to synchronous speed.
- (ii) Since the rotor has no teeth or salient poles or winding, a hysteresis motor is inherently quiet and produces smooth rotation of the load.



Fig.(9.25)

(iii) The rotor takes on the same number of poles as the stator field. Thus by changing the number of stator poles through pole-changing connections, we can get a set of synchronous speeds for the motor.

#### Applications

Due to their quiet operation and ability to drive high-inertia toads, hysteresis motors are particularly well suited for driving (i) electric clocks (ii) timing devices (iii) tape-decks (iv)from-tables and other precision audio-equipment.
# Chapter (11)

## **Synchronous Motors**

## Introduction

It may be recalled that a d.c. generator can be run as a d.c. motor. In like manner, an alternator may operate as a motor by connecting its armature winding to a 3-phase supply. It is then called a synchronous motor. As the name implies, a synchronous motor runs at synchronous speed ( $N_s = 120f/P$ ) i.e., in synchronism with the revolving field produced by the 3-phase supply. The speed of rotation is, therefore, tied to the frequency of the source. Since the frequency is fixed, the motor speed stays constant irrespective of the load or voltage of 3-phase supply. However, synchronous motors are not used so much because they run at constant speed (i.e., synchronous speed) but because they possess other unique electrical properties. In this chapter, we shall discuss the working and characteristics of synchronous motors.

## **11.1 Construction**

A synchronous motor is a machine that operates at synchronous speed and converts electrical energy into mechanical energy. It is fundamentally an alternator operated as a motor. Like an alternator, a synchronous motor has the following two parts:

- (i) a stator which houses 3-phase armature winding in the slots of the stator core and receives power from a 3-phase supply [See (Fig. (11.1)].
- (ii) a rotor that has a set of salient poles excited by direct current to form alternate N and S poles. The exciting coils are connected in series to two slip rings and direct current is fed into the winding from an external exciter mounted on the rotor shaft.

The stator is wound for the same number of poles as the rotor poles. As in the case of an induction motor, the number of poles determines the synchronous speed of the motor:



Synchronous speed,  $N_s = \frac{120f}{P}$ 

where f = frequency of supply in Hz P = number of poles

An important drawback of a synchronous motor is that it is not self-starting and auxiliary means have to be used for starting it.

#### 11.2 Some Facts about Synchronous Motor

Some salient features of a synchronous motor are:

- (i) A synchronous motor runs at synchronous speed or not at all. Its speed is constant (synchronous speed) at all loads. The only way to change its speed is to alter the supply frequency ( $N_s = 120$  f/P).
- (ii) The outstanding characteristic of a synchronous motor is that it can be made to operate over a wide range of power factors (lagging, unity or leading) by adjustment of its field excitation. Therefore, a synchronous motor can be made to carry the mechanical load at constant speed and at the same time improve the power factor of the system.
- (iii) Synchronous motors are generally of the salient pole type.
- (iv) A synchronous motor is not self-starting and an auxiliary means has to be used for starting it. We use either induction motor principle or a separate starting motor for this purpose. If the latter method is used, the machine must be run up to synchronous speed and synchronized as an alternator.

## **11.3 Operating Principle**

The fact that a synchronous motor has no starting torque can be easily explained.

- (i) Consider a 3-phase synchronous motor having two rotor poles  $N_R$  and  $S_R$ . Then the stator will also be wound for two poles  $N_S$  and  $S_S$ . The motor has direct voltage applied to the rotor winding and a 3-phase voltage applied to the stator winding. The stator winding produces a rotating field which revolves round the stator at synchronous speed  $N_s$ (= 120 f/P). The direct (or zero frequency) current sets up a two-pole field which is stationary so long as the rotor is not turning. Thus, we have a situation in which there exists a pair of revolving armature poles (i.e.,  $N_S S_S$ ) and a pair of stationary rotor poles (i.e.,  $N_R S_R$ ).
- (ii) Suppose at any instant, the stator poles are at positions A and B as shown in Fig. (11.2 (i)). It is clear that poles  $N_S$  and  $N_R$  repel each other and so do the poles  $S_S$  and  $S_R$ . Therefore, the rotor tends to move in the anticlockwise direction. After a period of half-cycle (or  $\frac{1}{2}$  f = 1/100 second), the polarities of the stator poles are reversed but the polarities of the rotor poles remain the same as shown in Fig. (11.2 (ii)). Now  $S_S$  and  $N_R$  attract

each other and so do  $N_S$  and  $S_R$ . Therefore, the rotor tends to move in the clockwise direction. Since the stator poles change their polarities rapidly, they tend to pull the rotor first in one direction and then after a period of half-cycle in the other. Due to high inertia of the rotor, the motor fails to start.



Hence, a synchronous motor has no self-starting torque i.e., a synchronous motor cannot start by itself.

How to get continuous unidirectional torque? If the rotor poles are rotated by some external means at such a speed that they interchange their positions along with the stator poles, then the rotor will experience a continuous unidirectional torque. This can be understood from the following discussion:

- (i) Suppose the stator field is rotating in the clockwise direction and the rotor is also rotated clockwise by some external means at such a speed that the rotor poles interchange their positions along with the stator poles.
- (ii) Suppose at any instant the stator and rotor poles are in the position shown in Fig. (11.3 (i)). It is clear that torque on the rotor will be clockwise. After a period of half-cycle, the stator poles reverse their polarities and at the same time rotor poles also interchange their positions as shown in Fig. (11.3 (ii)). The result is that again the torque on the rotor is clockwise. Hence a continuous unidirectional torque acts on the rotor and moves it in the clockwise direction. Under this condition, poles on the rotor always face poles of opposite polarity on the stator and a strong magnetic attraction is set up between them. This mutual attraction locks the rotor and stator together and the rotor is virtually pulled into step with the speed of revolving flux (i.e., synchronous speed).
- (iii) If now the external prime mover driving the rotor is removed, the rotor will continue to rotate at synchronous speed in the clockwise direction because the rotor poles are magnetically locked up with the stator poles. It is due to

this magnetic interlocking between stator and rotor poles that a synchronous motor runs at the speed of revolving flux i.e., synchronous speed.



**Fig.**(11.3)

## **11.4 Making Synchronous Motor Self-Starting**

A synchronous motor cannot start by itself. In order to make the motor self-starting, a squirrel cage winding (also called damper winding) is provided on the rotor. The damper winding consists of copper bars embedded in the pole faces of the salient poles of the rotor as shown in Fig. (11.4). The bars are short-circuited at the ends to form in effect a partial squirrel cage winding. The damper winding serves to start the motor.



- (i) To start with, 3-phase supply is given to the stator winding while the rotor field winding is left unenergized. The rotating stator field induces currents in the damper or squirrel cage winding and the motor starts as an induction motor.
- (ii) As the motor approaches the synchronous speed, the rotor is excited with direct current. Now the resulting poles on the rotor face poles of opposite polarity on the stator and a strong magnetic attraction is set up between them. The rotor poles lock in with the poles of rotating flux. Consequently, the rotor revolves at the same speed as the stator field i.e., at synchronous speed.
- (iii) Because the bars of squirrel cage portion of the rotor now rotate at the same speed as the rotating stator field, these bars do not cut any flux and, therefore, have no induced currents in them. Hence squirrel cage portion of the rotor is, in effect, removed from the operation of the motor.

It may be emphasized here that due to magnetic interlocking between the stator and rotor poles, a synchronous motor can only run at synchronous speed. At any other speed, this magnetic interlocking (i.e., rotor poles facing opposite polarity stator poles) ceases and the average torque becomes zero. Consequently, the motor comes to a halt with a severe disturbance on the line.

*Note*: It is important to excite the rotor with direct current at the right moment. For example, if the d.c. excitation is applied when N-pole of the stator faces N-pole of the rotor, the resulting magnetic repulsion will produce a violent mechanical shock. The motor will immediately slow down and the circuit breakers will trip. In practice, starters for synchronous motors arc designed to detect the precise moment when excitation should be applied.

## 11.5 Equivalent Circuit

Unlike the induction motor, the synchronous motor is connected to two electrical systems; a d.c. source at the rotor terminals and an a.c. system at the stator terminals.

- 1. Under normal conditions of synchronous motor operation, no voltage is induced in the rotor by the stator field because the rotor winding is rotating at the same speed as the stator field. Only the impressed direct current is present in the rotor winding and ohmic resistance of this winding is the only opposition to it as shown in Fig. (11.5 (i)).
- 2. In the stator winding, two effects are to be considered, the effect of stator field on the stator winding and the effect of the rotor field cutting the stator conductors at synchronous speed.



(i) The effect of stator field on the stator (or armature) conductors is accounted for by including an inductive reactance in the armature winding. This is called synchronous reactance  $X_s$ . A resistance  $R_a$  must be considered to be in series with this reactance to account for the copper losses in the stator or armature winding as shown in Fig. (11.5 (i)). This

resistance combines with synchronous reactance and gives the synchronous impedance of the machine.

(ii) The second effect is that a voltage is generated in the stator winding by the synchronously-revolving field of the rotor as shown in Fig. (11.5 (i)). This generated e.m.f.  $E_B$  is known as back e.m.f. and opposes the stator voltage V. The magnitude of  $E_b$  depends upon rotor speed and rotor flux  $\phi$  per pole. Since rotor speed is constant; the value of  $E_b$  depends upon the rotor flux per pole i.e. exciting rotor current  $I_f$ .

Fig. (11.5 (i)) shows the schematic diagram for one phase of a star-connected synchronous motor while Fig. (11.5 (ii)) shows its equivalent circuit. Referring to the equivalent circuit in Fig. (11.5 (ii)).

Net voltage/phase in stator winding is

 $Z_s = \sqrt{R_a^2 + X_s^2}$ 

$$E_r = V - E_b$$
 phasor difference

Armature current/phase,  $I_a = \frac{E_r}{Z_s}$ 

where

This equivalent circuit helps considerably in understanding the operation of a synchronous motor.

A synchronous motor is said to be normally excited if the field excitation is such that  $E_b = V$ . If the field excitation is such that  $E_b < V$ , the motor is said to be under-excited. The motor is said to be over-excited if the field excitation is such that  $E_b > V$ . As we shall see, for both normal and under excitation, the motor has lagging power factor. However, for over-excitation, the motor has leading power factor.

*Note*: In a synchronous motor, the value of  $X_s$  is 10 to 100 times greater than  $R_a$ . Consequently, we can neglect  $R_a$  unless we are interested in efficiency or heating effects.

## **11.6 Motor on Load**

In d.c. motors and induction motors, an addition of load causes the motor speed to decrease. The decrease in speed reduces the counter e.m.f. enough so that additional current is drawn from the source to carry the increased load at a reduced speed. This action cannot take place in a synchronous motor because it runs at a constant speed (i.e., synchronous speed) at all loads.

What happens when we apply mechanical load to a synchronous motor? The rotor poles fall slightly behind the stator poles while continuing to run at

synchronous speed. The angular displacement between stator and rotor poles (called torque angle  $\alpha$ ) causes the phase of back e.m.f.  $E_b$  to change w.r.t. supply voltage V. This increases the net e.m.f.  $E_r$  in the stator winding. Consequently, stator current  $I_a$  ( $= E_r/Z_s$ ) increases to carry the load.



**Fig.(11.6)** 

The following points may be noted in synchronous motor operation:

- (i) A synchronous motor runs at synchronous speed at all loads. It meets the increased load not by a decrease in speed but by the relative shift between stator and rotor poles i.e., by the adjustment of torque angle  $\alpha$ .
- (ii) If the load on the motor increases, the torque angle a also increases (i.e., rotor poles lag behind the stator poles by a greater angle) but the motor continues to run at synchronous speed. The increase in torque angle  $\alpha$  causes a greater phase shift of back e.m.f.  $E_b$  w.r.t. supply voltage V. This increases the net voltage  $E_r$  in the stator winding. Consequently, armature current  $I_a$  (=  $E_r/Z_s$ ) increases to meet the load demand.
- (iii) If the load on the motor decreases, the torque angle  $\alpha$  also decreases. This causes a smaller phase shift of  $E_b$  w.r.t. V. Consequently, the net voltage  $E_r$  in the stator winding decreases and so does the armature current  $I_a$  (=  $E_r/Z_s$ ).

## **11.7 Pull-Out Torque**

There is a limit to the mechanical load that can be applied to a synchronous motor. As the load increases, the torque angle  $\alpha$  also increases so that a stage is reached when the rotor is pulled out of synchronism and the motor comes to a standstill. This load torque at which the motor pulls out of synchronism is called pull—out or breakdown torque. Its value varies from 1.5 to 3.5 times the full—load torque.

When a synchronous motor pulls out of synchronism, there is a major disturbance on the line and the circuit breakers immediately trip. This protects the motor because both squirrel cage and stator winding heat up rapidly when the machine ceases to run at synchronous speed.

## **11.8 Motor Phasor Diagram**

Consider an under-excited ^tar-connected synchronous motor ( $E_b < V$ ) supplied with fixed excitation i.e., back e.m.f.  $E_b$  is constant-

Let V = supply voltage/phase  $E_b =$  back e.m.f./phase  $Z_s =$  synchronous impedance/phase

#### (i) Motor on no load

When the motor is on no load, the torque angle  $\alpha$  is small as shown in Fig. (11.7 (i)). Consequently, back e.m.f.  $E_b$  lags behind the supply voltage V by a small angle  $\delta$  as shown in the phasor diagram in Fig. (11.7 (iii)). The net voltage/phase in the stator winding, is  $E_r$ .

Armature current/phase,  $I_a = E_r/Z_s$ 

The armature current  $I_a$  lags behind  $E_r$  by  $\theta = \tan^{-1} X_s/R_a$ . Since  $X_s >> R_a$ ,  $I_a$  lags  $E_r$  by nearly 90°. The phase angle between V and  $I_a$  is  $\phi$  so that motor power factor is  $\cos \phi$ .

Input power/phase = V  $I_a \cos \phi$ 



Thus at no load, the motor takes a small power  $VI_a \cos \phi$ /phase from the supply to meet the no-load losses while it continues to run at synchronous speed.

#### (ii) Motor on load

When load is applied to the motor, the torque angle a increases as shown in Fig. (11.8 (i)). This causes  $E_b$  (its magnitude is constant as excitation is fixed) to lag behind V by a greater angle as shown in the phasor diagram in Fig. (11.8 (ii)). The net voltage/phase  $E_r$  in the stator winding increases. Consequently, the motor draws more armature current  $I_a$  (= $E_r/Z_s$ ) to meet the applied load. Again  $I_a$  lags  $E_r$  by about 90° since  $X_s >> R_a$ . The power factor of the motor is

Again  $I_a$  lags  $E_r$  by about 90° since  $X_s >> R_a$ . The power factor of the motor is  $\cos \phi$ .

Input power/phase,  $P_i = V I_a \cos \phi$ 

Mechanical power developed by motor/phase

 $P_m = E_b \times I_a \times cosine$  of angle between  $E_b$  and  $I_a$ 



**Fig.(11.8)** 

## 11.9 Effect of Changing Field Excitation at Constant Load

In a d.c. motor, the armature current  $I_a$  is determined by dividing the difference between V and  $E_b$  by the armature resistance  $R_a$ . Similarly, in a synchronous motor, the stator current ( $I_a$ ) is determined by dividing voltage-phasor resultant ( $E_r$ ) between V and  $E_b$  by the synchronous impedance  $Z_s$ .

One of the most important features of a synchronous motor is that by changing the field excitation, it can be made to operate from lagging to leading power factor. Consider a synchronous motor having a fixed supply voltage and driving a constant mechanical load. Since the mechanical load as well as the speed is constant, the power input to the motor (=3 VI<sub>a</sub> cos  $\phi$ ) is also constant. This means that the in-phase component I<sub>a</sub> cos  $\phi$  drawn from the supply will remain constant. If the field excitation is changed, back e.m.f E<sub>b</sub> also changes. This results in the change of phase position of I<sub>a</sub> w.r.t. V and hence the power factor cos  $\phi$  of the motor changes. Fig. (11.9) shows the phasor diagram of the synchronous motor for different values of field excitation. Note that extremities of current phasor I<sub>a</sub> lie on the straight line AB.

#### (i) Under excitation

The motor is said to be under-excited if the field excitation is such that  $E_b < V$ . Under such conditions, the current  $I_a$  lags behind V so that motor power factor is lagging as shown in Fig. (11.9 (i)). This can be easily explained. Since  $E_b < V$ , the net voltage  $E_r$  is decreased and turns clockwise. As angle  $\theta$  (= 90°) between  $E_r$  and  $I_a$  is constant, therefore, phasor  $I_a$  also turns clockwise i.e., current  $I_a$  lags behind the supply voltage. Consequently, the motor has a lagging power factor.

#### (ii) Normal excitation

The motor is said to be normally excited if the field excitation is such that  $E_b = V$ . This is shown in Fig. (11.9 (ii)). Note that the effect of increasing excitation (i.e., increasing  $E_b$ ) is to turn the phasor  $E_r$  and hence  $I_a$  in the anti-clockwise direction i.e.,  $I_a$  phasor has come closer to phasor V. Therefore, p.f. increases though still lagging. Since input power (=3 V  $I_a \cos \phi$ ) is unchanged, the stator current  $I_a$  must decrease with increase in p.f.



**Fig.(11.9)** 

Suppose the field excitation is increased until the current  $I_a$  is in phase with the applied voltage V, making the p.f. of the synchronous motor unity [See Fig. (11.9 (iii))]. For a given load, at unity p.f. the resultant  $E_r$  and, therefore,  $I_a$  are minimum.

#### (iii) Over excitation

The motor is said to be overexcited if the field excitation is such that  $E_b > V$ . Under-such conditions, current  $I_a$  leads V and the motor power factor is leading as shown in Fig. (11.9 (iv)). Note that  $E_r$  and hence  $I_a$  further turn anti-clockwise from the normal excitation position. Consequently,  $I_a$  leads V.

From the above discussion, it is concluded that if the synchronous motor is under-excited, it has a lagging power factor. As the excitation is increased, the power factor improves till it becomes unity at normal excitation. Under such conditions, the current drawn from the supply is minimum. If the excitation is further increased (i.e., over excitation), the motor power factor becomes leading.

*Note*. The armature current  $(I_a)$  is minimum at unity p.f and increases as the power factor becomes poor, either leading or lagging.

## **11.10Phasor Diagrams With Different Excitations**

Fig. (11.10) shows the phasor diagrams for different field excitations at constant load. Fig. (11.10 (i)) shows the phasor diagram for normal excitation ( $E_b = V$ ), whereas Fig. (11.10 (ii)) shows the phasor diagram for under-excitation. In both cases, the motor has lagging power factor.

Fig. (11.10 (iii)) shows the phasor diagram when field excitation is adjusted for unity p.f. operation. Under this condition, the resultant voltage  $E_r$  and, therefore, the stator current  $I_a$  are minimum. When the motor is overexcited, it has leading power factor as shown in Fig. (11.10 (iv)). The following points may be remembered:

- (i) For a given load, the power factor is governed by the field excitation; a weak field produces the lagging armature current and a strong field produces a leading armature current.
- (ii) The armature current  $(I_a)$  is minimum at unity p.f and increases as the p.f. becomes less either leading or lagging.



Fig.(11.10)

## **11.11 Power Relations**

Consider an under-excited star-connected synchronous motor driving a mechanical load. Fig. (11.11 (i)) shows the equivalent circuit for one phase, while Fig. (11.11 (ii)) shows the phasor diagram.



**Fig.**(11.11)

- (i) Input power/phase,  $P_i = V I_a \cos \phi$
- (ii) Mechanical power developed by the motor/phase,

 $P_m = E_b \times I_a \times cosine$  of angle between  $E_b$  and  $I_a$ 

 $= E_b I_a \cos(\delta - \phi)$ 

- (iii) Armature Cu loss/phase =  $I_a^2 R_a = P_i P_m$
- (iv) Output power/phasor,  $P_{out} = P_m Iron$ , friction and excitation loss.

Fig. (11.12) shows the power flow diagram of the synchronous motor.





### **11.12Motor Torque**

Gross torque, 
$$T_g = 9.55 \frac{P_m}{N_s}$$
 N - m

where

 $P_m$  = Gross motor output in watts =  $E_b I_a \cos(\delta - \phi)$ N<sub>s</sub> = Synchronous speed in r.p.m.

Shaft torque, 
$$T_{sh} = 9.55 \frac{P_{out}}{N_s}$$
 N - m

It may be seen that torque is directly proportional to the mechanical power because rotor speed (i.e.,  $N_s$ ) is fixed.

### **11.13 Mechanical Power Developed By Motor**

(Armature resistance neglected) Fig. (11.13) shows the phasor diagram of an under-excited synchronous motor driving a mechanical load. Since armature resistance  $R_a$  is assumed zero.  $\tan \theta = X_s/R_a = \infty$  and hence  $\theta = 90^\circ$ .

Input power/phase = V  $I_a \cos \phi$ 



**Fig.**(11.13)

Since  $R_a$  is assumed zero, stator Cu loss  $(I_a^2 R_a)$  will be zero. Hence input power is equal to the mechanical power  $P_m$  developed by the motor.

Mech. power developed/ phase,  $P_m = V I_a \cos \phi$  (i)

Referring to the phasor diagram in Fig. (11.13),

 $AB = E_r \cos \phi = I_a X_s \cos \phi$ 

Also

$$AB = E_b \sin \delta$$

$$: \qquad E_b \sin \delta = I_a X_s \cos \phi$$

or

$$I_a \cos \phi = \frac{E_b \sin \delta}{X_s}$$

Substituting the value of  $I_a \cos \phi$  in exp. (i) above,

$$P_{\rm m} = \frac{VE_{\rm b}}{X_{\rm s}} \quad \text{per phase}$$
$$= \frac{VE_{\rm b}}{X_{\rm s}} \quad \text{for 3-phase}$$

It is clear from the above relation that mechanical power increases with torque angle (in electrical degrees) and its maximum value is reached when  $\delta = 90^{\circ}$  (electrical).

$$P_{max} = \frac{VE_b}{X_s}$$
 per phase

Under this condition, the poles of the rotor will be mid-way between N and S poles of the stator.

#### **11.14** Power Factor of Synchronous Motors

In an induction motor, only one winding (i.e., stator winding) produces the necessary flux in the machine. The stator winding must draw reactive power from the supply to set up the flux. Consequently, induction motor must operate at lagging power factor.

But in a synchronous motor, there are two possible sources of excitation; alternating current in the stator or direct current in the rotor. The required flux may be produced either by stator or rotor or both.

(i) If the rotor exciting current is of such magnitude that it produces all the required flux, then no magnetizing current or reactive power is needed in the stator. As a result, the motor will operate at unity power factor.

- (ii) If the rotor exciting current is less (i.e., motor is under-excited), the deficit in flux is made up by the stator. Consequently, the motor draws reactive power to provide for the remaining flux. Hence motor will operate at a lagging power factor.
- (iii) If the rotor exciting current is greater (i.e., motor is over-excited), the excess flux must be counterbalanced in the stator. Now the stator, instead of absorbing reactive power, actually delivers reactive power to the 3-phase line. The motor then behaves like a source of reactive power, as if it were a capacitor. In other words, the motor operates at a leading power factor.

To sum up, a synchronous motor absorbs reactive power when it is underexcited and delivers reactive power to source when it is over-excited.

## **11.15 Synchronous Condenser**

A synchronous motor takes a leading current when over-excited and, therefore, behaves as a capacitor.

An over-excited synchronous motor running on no-load in known as synchronous condenser.

When such a machine is connected in parallel with induction motors or other devices that operate at low lagging power factor, the leading kVAR supplied by the synchronous condenser partly neutralizes the lagging reactive kVAR of the loads. Consequently, the power factor of the system is improved.

Fig. (11.14) shows the power factor improvement by synchronous condenser method. The  $3 - \phi$  load takes current  $I_L$  at low lagging power factor  $\cos \phi_L$ . The synchronous condenser takes a current  $I_m$  which leads the voltage by an angle  $\phi_m$ . The resultant current I is the vector sum of  $I_m$  and  $I_L$  and lags behind the voltage by an angle  $\phi$ . It is clear that  $\phi$  is less than  $\phi_L$  so that  $\cos \phi$  is greater than  $\cos \phi_L$ . Thus the power factor is increased from  $\cos \phi_L$  to  $\cos \phi$ . Synchronous condensers are generally used at major bulk supply substations for power factor improvement.

#### Advantages

- (i) By varying the field excitation, the magnitude of current drawn by the motor can be changed by any amount. This helps in achieving stepless control of power factor.
- (ii) The motor windings have high thermal stability to short circuit currents.
- (iii) The faults can be removed easily.





#### Disadvantages

- (i) There are considerable losses in the motor.
- (ii) The maintenance cost is high.
- (iii) It produces noise.
- (iv) Except in sizes above 500 RVA, the cost is greater than that of static capacitors of the same rating.
- (v) As a synchronous motor has no self-starting torque, then-fore, an auxiliary equipment has to be provided for this purpose.

## **11.16** Applications of Synchronous Motors

- (i) Synchronous motors are particularly attractive for low speeds (< 300 r.p.m.) because the power factor can always be adjusted to unity and efficiency is high.
- (ii) Overexcited synchronous motors can be used to improve the power factor of a plant while carrying their rated loads.
- (iii) They are used to improve the voltage regulation of transmission lines.
- (iv) High-power electronic converters generating very low frequencies enable us to run synchronous motors at ultra-low speeds. Thus huge motors in the 10 MW range drive crushers, rotary kilns and variable-speed ball mills.

S. No.	Particular	Synchronous Motor	3-phase Induction Motor
1.	Speed	Remains constant (i.e., N <sub>s</sub> ) from no-load to full-load.	Decreases with load.
2.	Power factor	Can be made to operate from lagging to leading power factor.	Operates at lagging power factor.
3.	Excitation	Requires d.c. excitation at the rotor.	No excitation for the rotor.
4.	Economy	Economical fcr speeds below 300 r.p.m.	Economical for speeds above 600 r.p.m.
5.	Self-starting	No self-starting torque. Auxiliary means have to be provided for starting.	Self-starting
6.	Construction	Complicated	Simple
7.	Starting torque	More	less

## **11.17** Comparison of Synchronous and Induction Motors